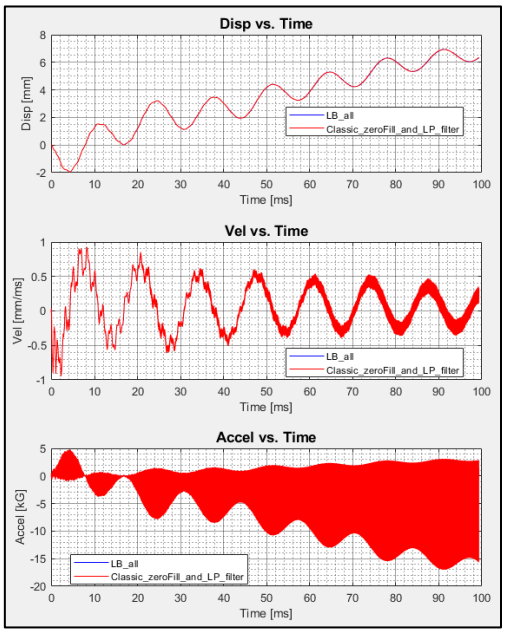
BODIE TECHNOLOGY, INC.
*The Noisy Data Experts™*

# Enhanced and Accurate Upsampling for Derivative-Sensitive Shock Signals

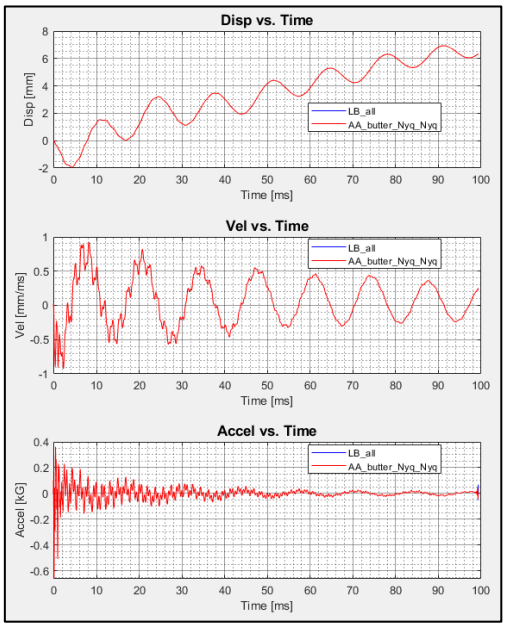
**Ted Diehl, PhD, Bodie Technology, Inc**

**96th Shock & Vibration Symposium, 2026**




How to avoid this

and instead get this




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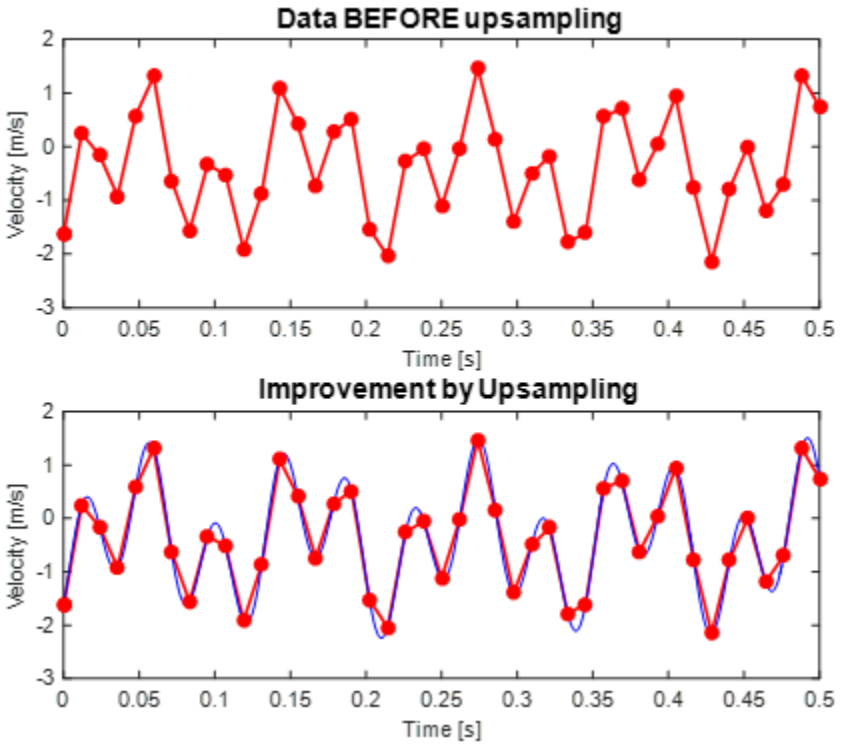
# Overview

- Upsampling – What is it and Why is it Important?
- Upsampling is a Key Component in Split-Band Filtering
- Exemplar Signal to Study
  - LP filtered Displacement and a decimated representation of the same signal with nearly equivalent frequency content.
- Upsampling Challenge & Upsampling Algorithms Considered
  1. Classic FFT Zero-Pad
  2. Classic Zero-Fill and LP Filter
  3. Kornucopia's Prior Upsampling
  4. Kornucopia's New Enhanced Upsampling (Version 5.3-05)

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# Upsampling – What is it and Why is it Important?

- **Upsampling**
  - The act of ***increasing*** the sampling rate of a signal.
    - Theoretically this should be done without adding any frequency content beyond the Nyquist frequency of the original sampling rate.
- **Upsampling scenarios**
  - Increase the sampling of a coarse but unaliased signal to achieve greater time-domain fidelity (more accurately estimate peak values and improve the visual time-domain interpretation).
  - Increase sampling rate to compute SRS or PVSS far enough out in natural frequency to fully cover all frequency content of driving acceleration.
  - As part of a split-band filtering algorithm.




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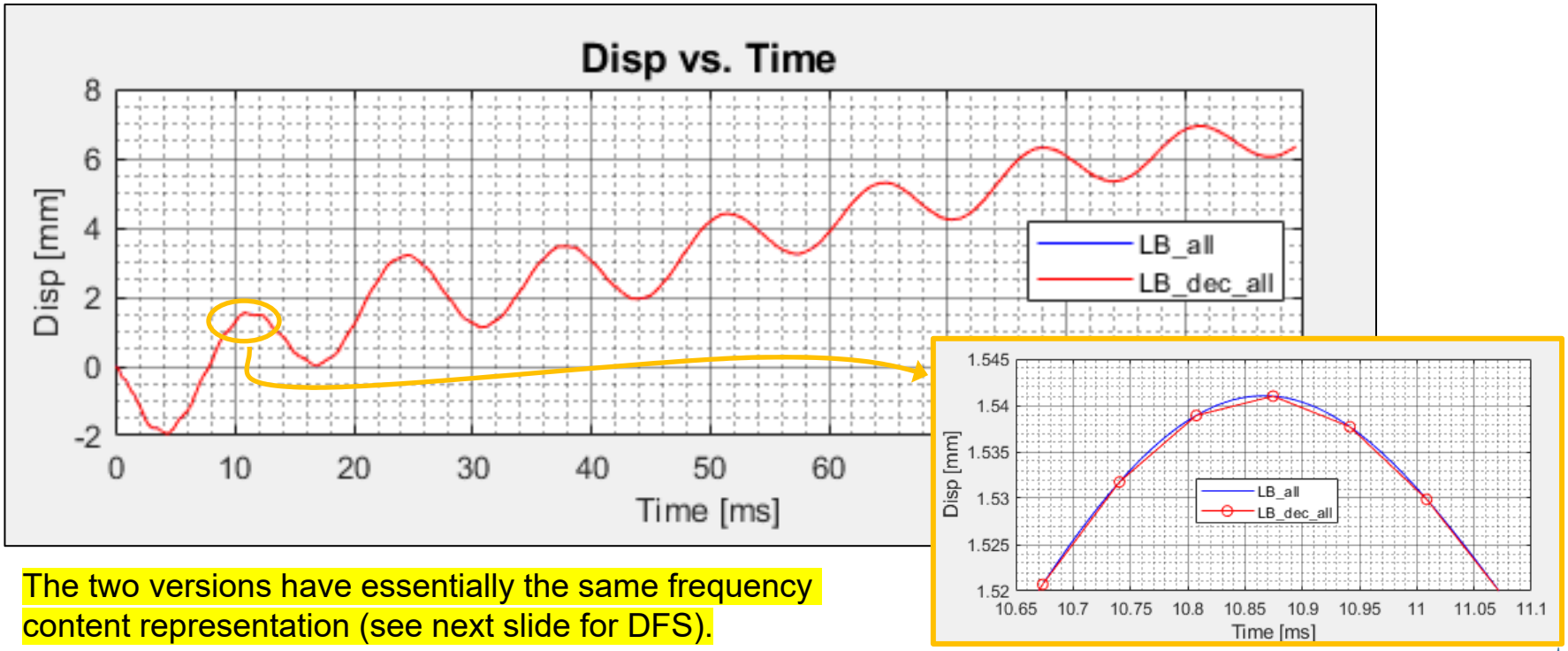
# Upsampling is a Key Component in Split-Band Filtering

- Split-Band filtering is a method to efficiently and stably process highly sampled data with filters that have extremely low cutoff frequencies relative to the data sampling frequency.
- Typical severe shock example:
  - Raw sampling rate of signal: fs = 1*MSa/s or higher.
  - Want to apply very low HP filter to remove drifting.
    - fc_HP = 20*Hz (or lower, perhaps as low as 2*Hz or even lower).
    - Ratio of fc_HP / fs = 2.0e-5 or smaller.
      - This commonly leads to ill-conditioned digital filtering.
  - **Solution to prevent ill-conditioning:** ***Split-band filtering using cascaded filters.***
    - Using an LP filter, divide the signal into Upper and Lower frequency bands.
      - The drifting will be in the Lower band.
    - Use decimation on the Lower band to improve numeric stability.
      - If needed, multiple levels of LP filtering and decimation may be needed.
      - Once ratio of fc_HP / fs_dec is around 1e-3 or larger, you are generally in a numerically stable range to apply the HP filter.
    - Re-assemble the levels by upsampling the Lower levels back to the Upper levels.

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# Exemplar Signal to Study

- We start with the LB (Low-Band) portion of a signal from a split-band calculation.
  - The transient Displacement has two representations:
    - **Original 1*MSa/s sampling**, but LP filtered with fc = 5.284*kHz.
    - **Decimated version**: fs_dec = 14.92*kSa/s (R = decFactor ≈ 67)
      - AA decimation LP filter used fc = 5.284*kHz (ensures 1e-3 gain @ Nyq).



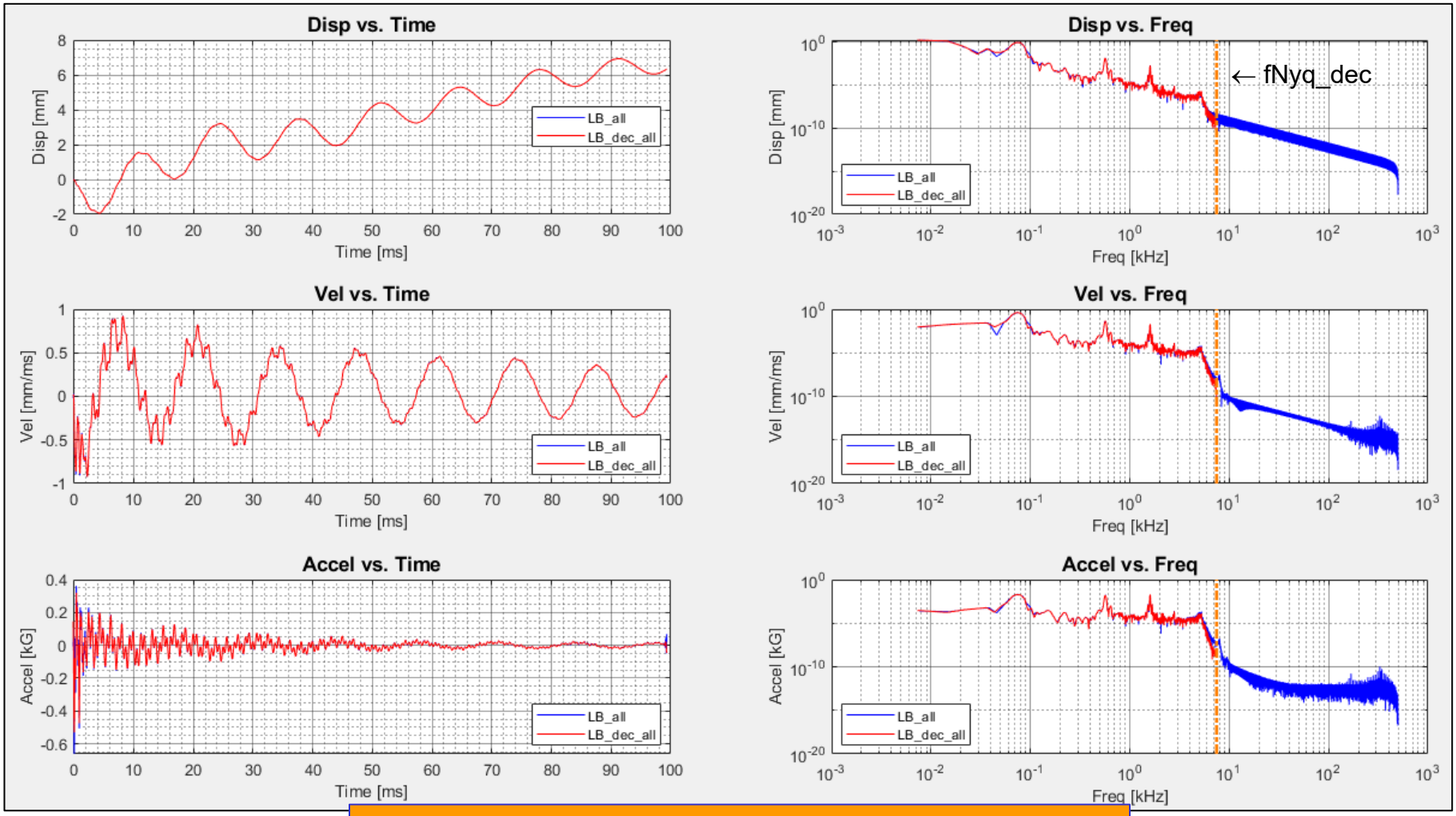

The two versions have essentially the same frequency content representation (see next slide for DFS).

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# Starting the Assessment

Numerical derivatives utilized advanced spline-based algorithms.

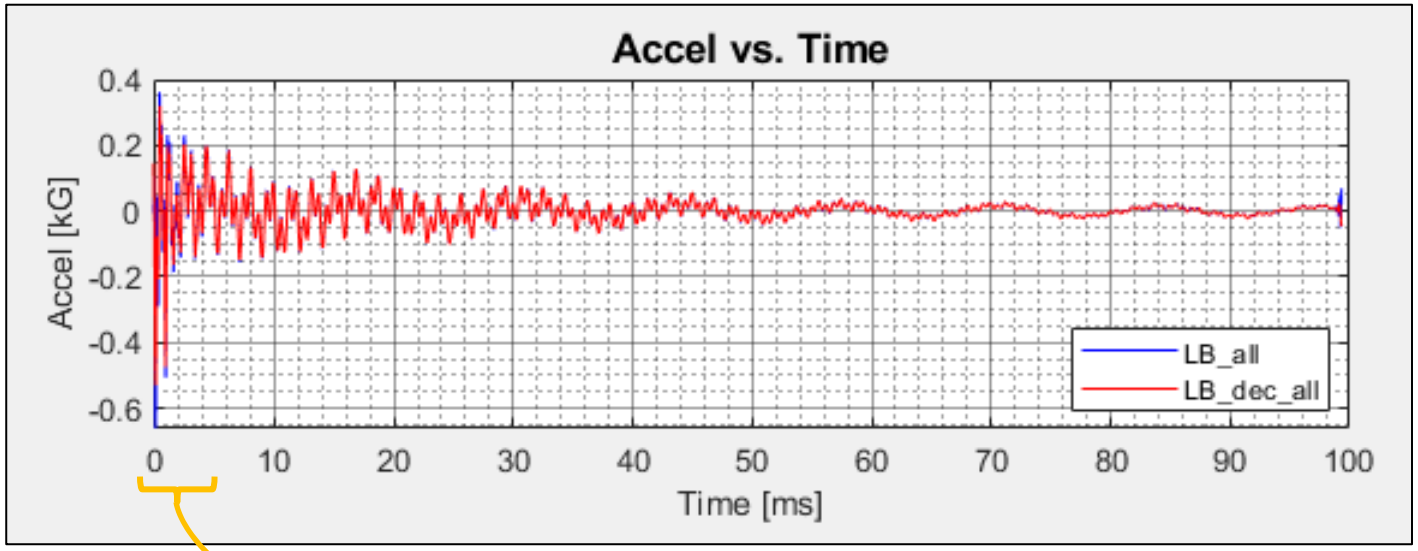
- The assessment starts with the **Displacement form**. Both the undecimated and decimated signals are then numerically differentiated to yield **Velocity** & **Acceleration** forms.



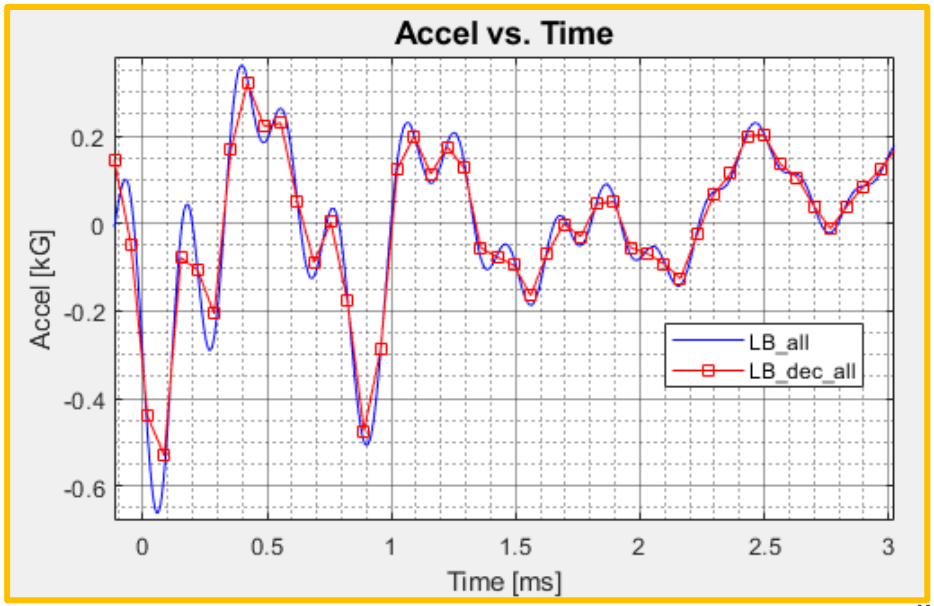

Kinematics of undecimated and decimated forms track excellently. These are our benchmarks.

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# Closer Look at the Acceleration Signals




Accel computed from decimated Displacement matches well with Accel computed from undecimated Disp.




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# Upsampling Challenge & Upsampling Algorithms Considered

- **The Challenge**
  1. Upsample the decimated Displacement signal to the same sampling rate as undecimated Displacement
  2. Then show that re-computing Velocity and Acceleration from the upsampled signal yields accurate results.
     - Sounds simple enough … Let's see!
- **Upsampling Algorithms to Assess:**
  1. **Classic FFT Zero-Pad**
     - Via MATLAB's `interpft` function.
  2. **Classic Zero-Fill and LP Filter**
     - Via MATLAB's `interp` function.
  3. **Kornucopia's Prior Upsampling**
     - Via `k_upsample` function using a classic zero-fill and LP filter algorithm based on a symmetric FIR-blackman filter.
  4. **Kornucopia's New Enhanced Upsampling (Version 5.3-05)**
     - Via the improved `k_upsample` function using a bi-directional Butterworth AA filter applied to the zero-fill and LP filter method.

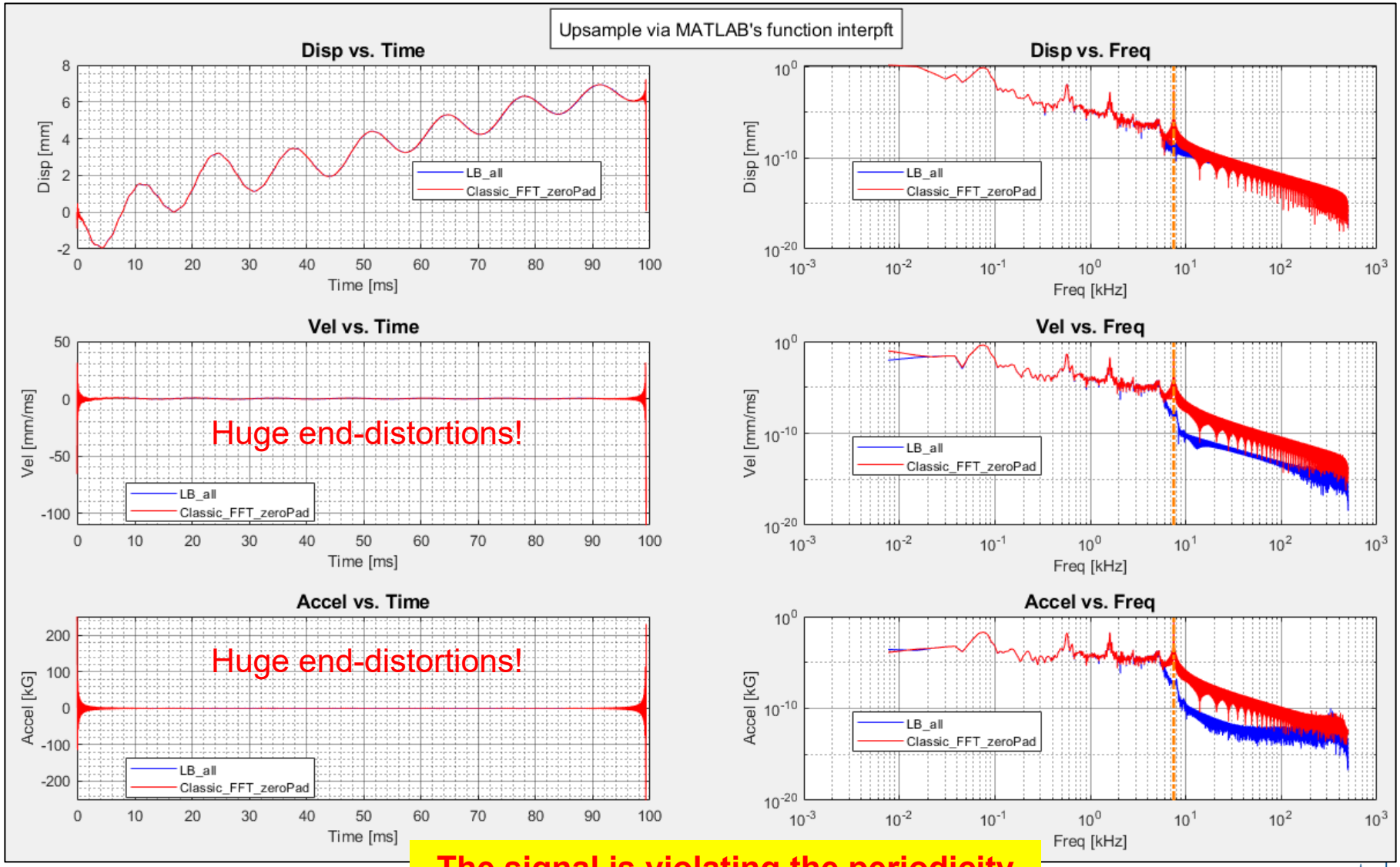
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# Classic FFT Zero-Pad Approach

- **Basic algorithm steps for the FFT Zero-Pad Approach**
  1. Compute FFT kernel of the time-domain Displacement signal to be upsampled.
  2. Zero-pad the complex coefficient vector to make the FFT think the bandwidth is much wider, but the additional frequency content has zero magnitude values.
  3. Compute inverse FFT kernel from zero-padded coefficients to achieve upsampled time-domain representation.
  - The 3 steps above are all handled via MATLAB's **`interpft`** function.
  - **A key point to understand**
    - The internal FFT kernel function assumes and imposes that the finite-length vector of data is interpreted as a periodic signal.
      - If the signal does not satisfy this assumption, strange end-effect distortions will occur!
- **For our assessment:**
  - The above calculations yield the upsampled Displacement signal.
  - We additionally compute Velocity and Acceleration using Kornucopia's `k_derivative` function, which utilizes advanced spline-based algorithms.

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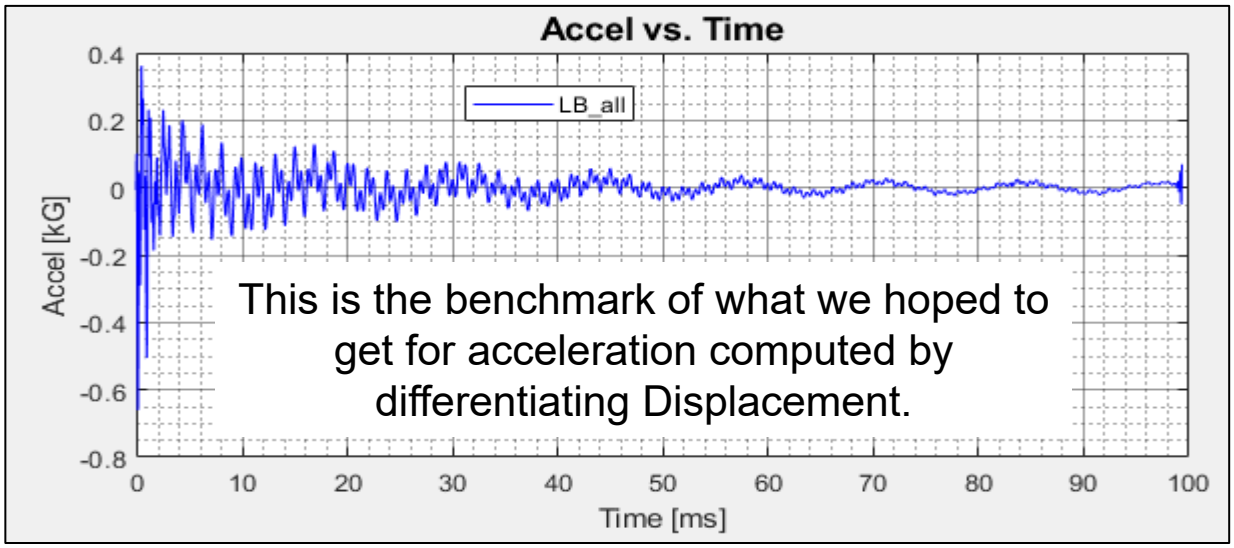
# Classic FFT Zero-Pad Approach – Results are Terrible




**The signal is violating the periodicity assumption and killing our results!**

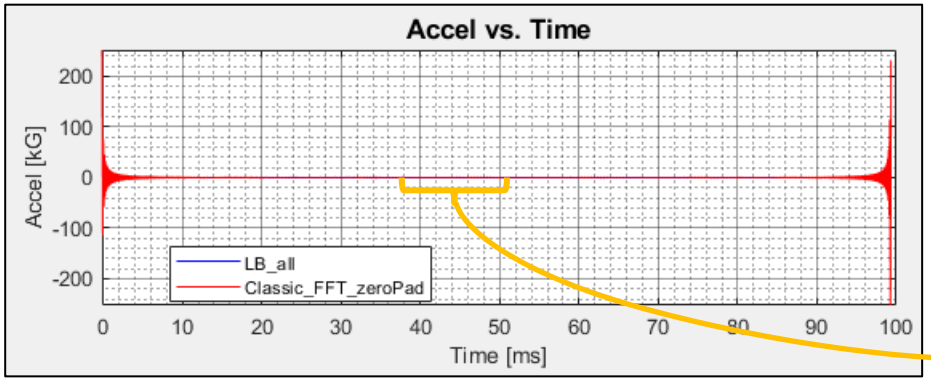
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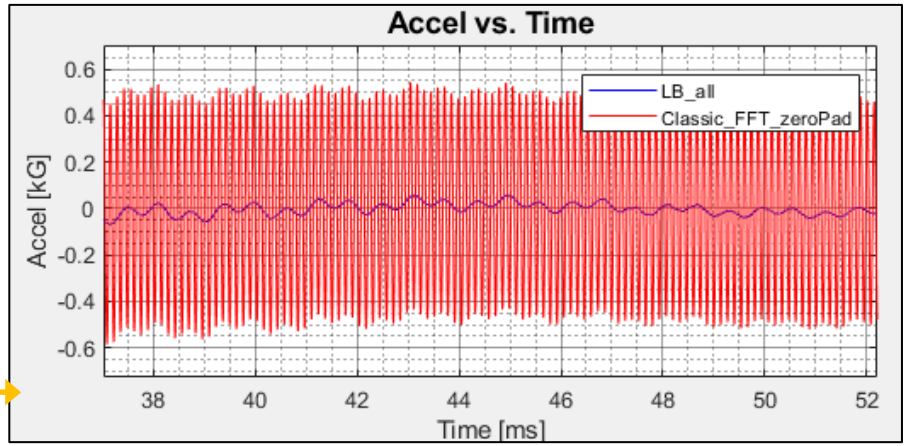
# Classic FFT Zero-Pad Approach – More on the Poor Results




Below is what we found using the **Classic FFT Zero-Pad.**

**The signal is violating the periodicity assumption and yields a disaster!**



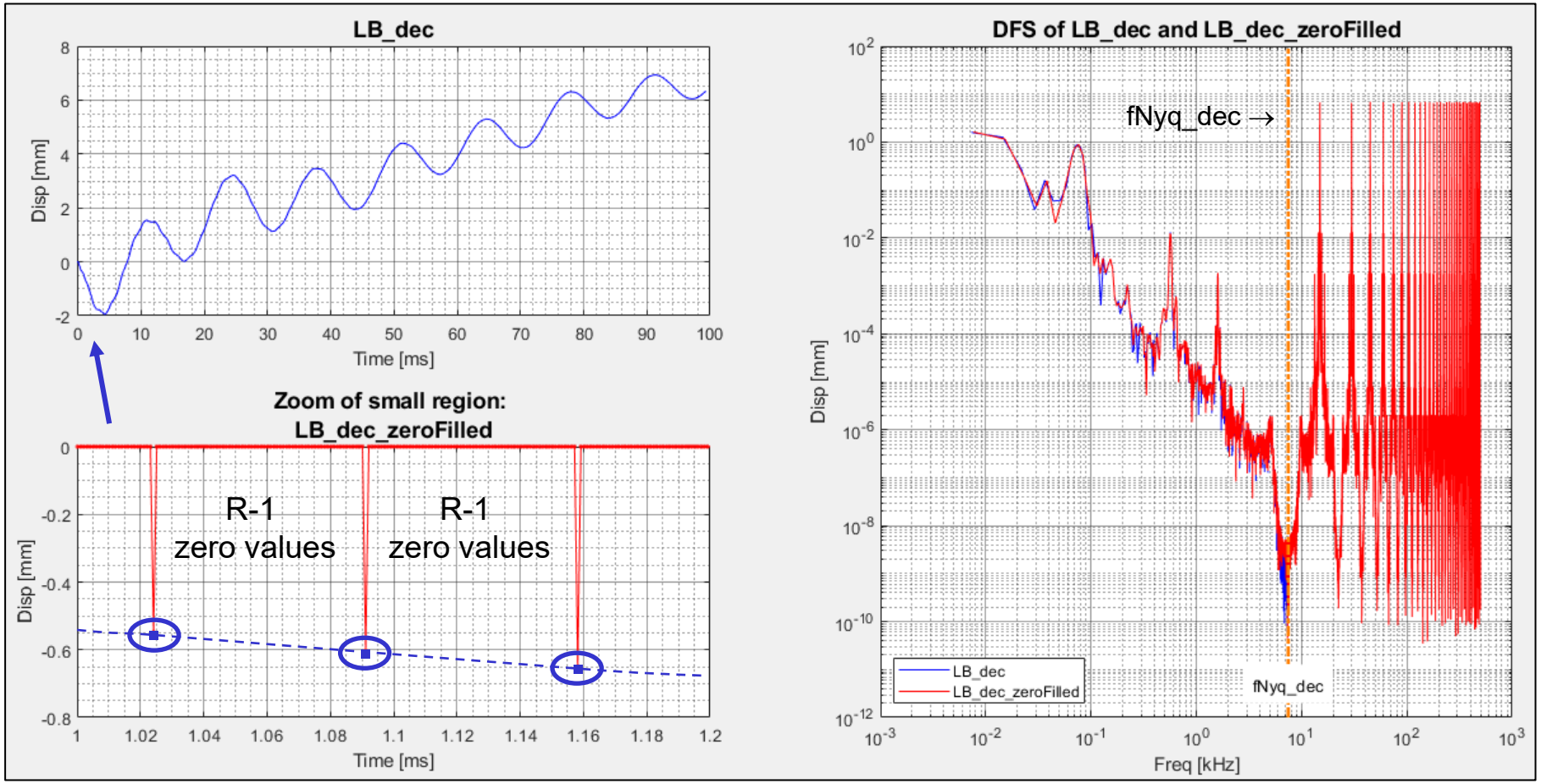

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# Upsampling via a Classic Zero-Fill and LP Filter Approach

- **Basic algorithm steps for the Classic Zero-Fill and LP Filter Approach**
  1. For an upsampling factor of R, insert R-1 zero amplitude values between each actual value of the signal to upsample.
  2. Apply a specially designed LP filter with the intent to allow the original data to pass through unchanged and interpolate the zero-filled values to minimize the mean-square error between the interpolated points and their ideal values.
     - This specially designed filter is typically a symmetric FIR filter.
     - Along with yielding interpolated results, the LP filter *should* be attenuating any out-of-band energy beyond the Nyquist frequency associated with the original, pre-upsampling sample rate.
  - The 2 steps above are all handled via MATLAB's **`interp`** function.
- **For our assessment:**
  - The above calculations yield the upsampled Displacement signal.
  - We additionally compute Velocity and Acceleration using Kornucopia's `k_derivative` function, which utilizes advanced spline-based algorithms.

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# Look Just at the *Zero-Fill Portion* of the Zero-Fill and LP Filter Approach

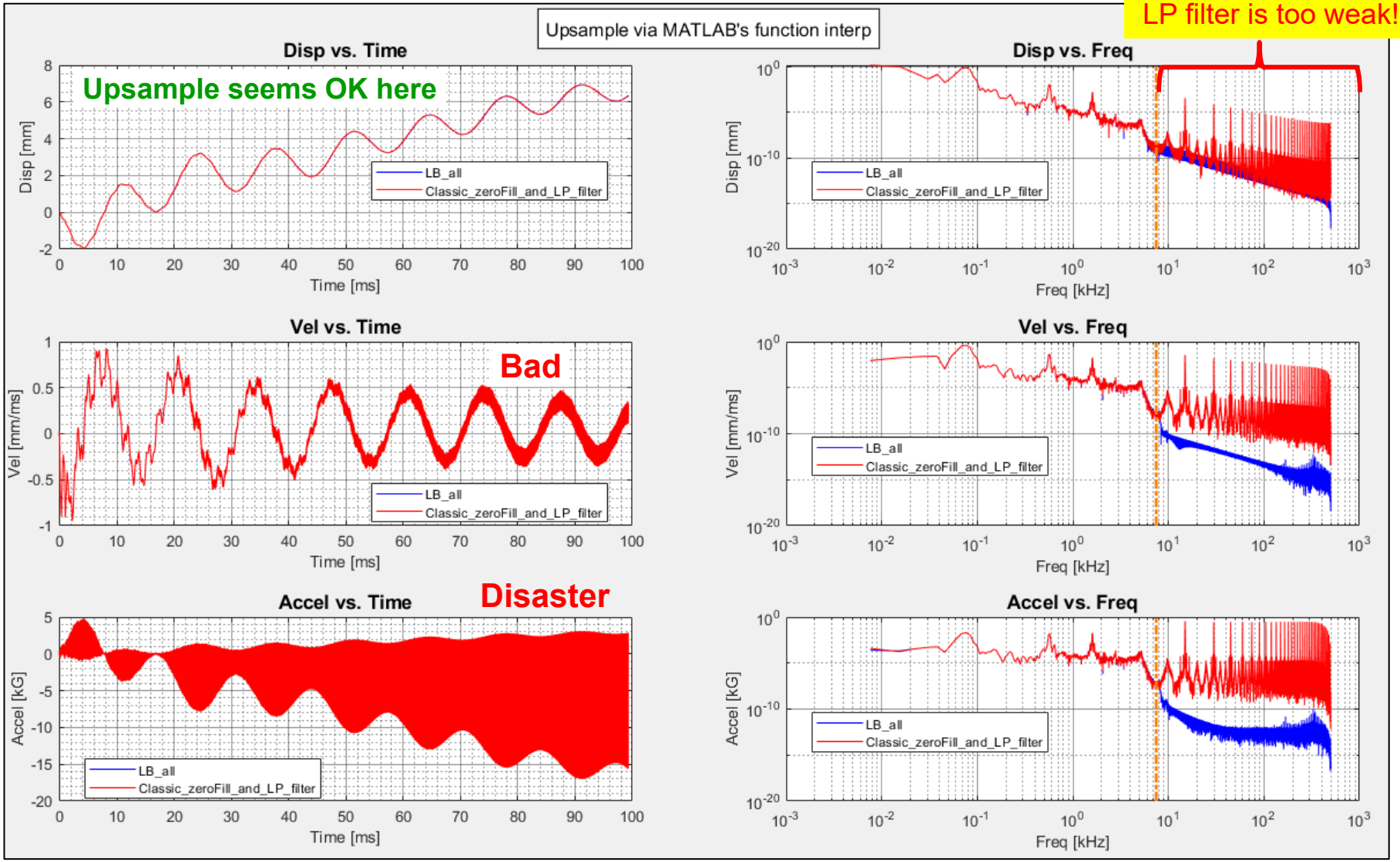



**These 3 points are from the data to be upsampled**

The huge number of spectral replicas beyond fNyq_dec is caused by the zero-filling. ***Ideally, the LP filter should attenuate ALL of this false content.***

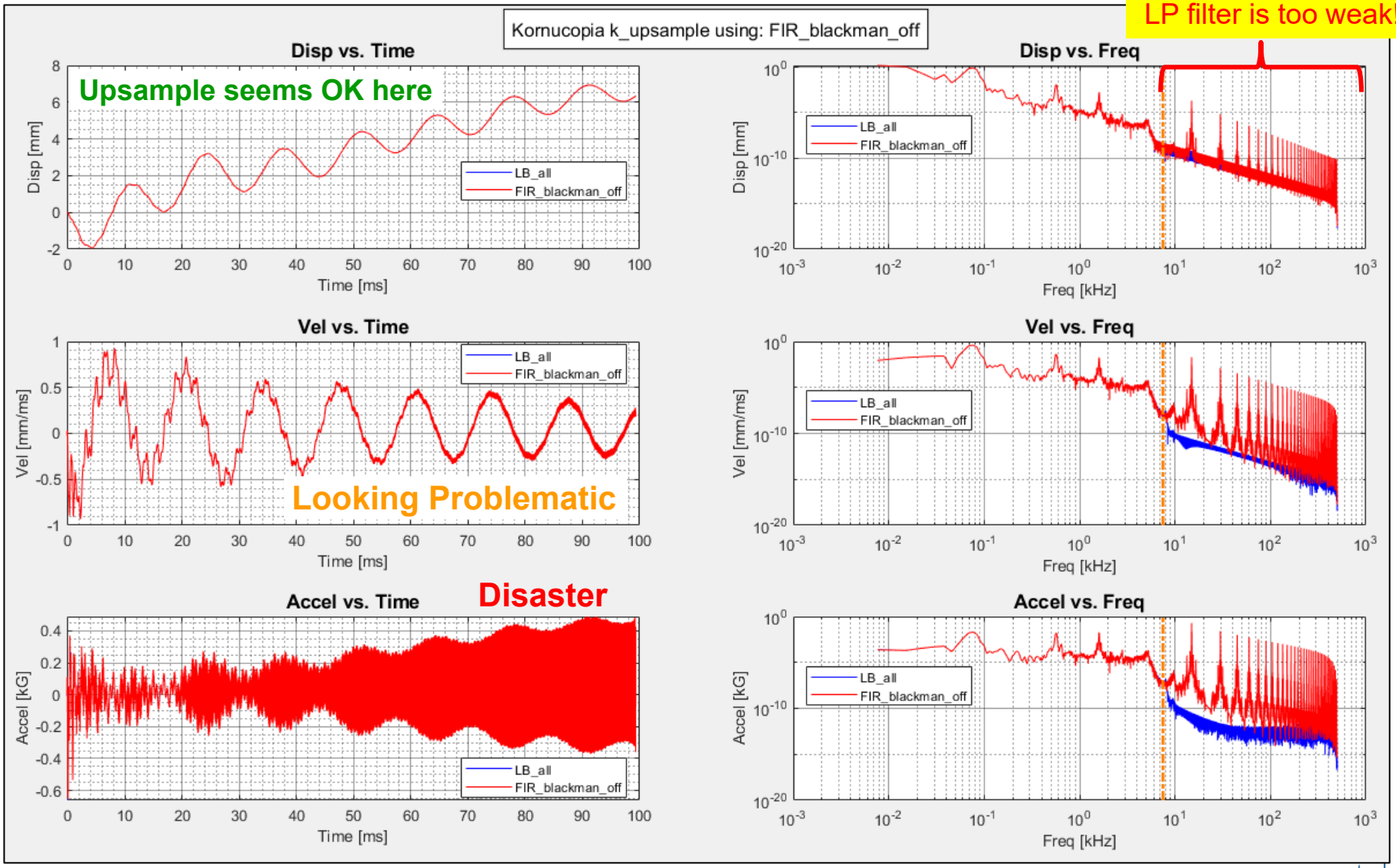
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# Classic Zero-Fill and LP Filter Upsampling via MATLAB's `interp` Function




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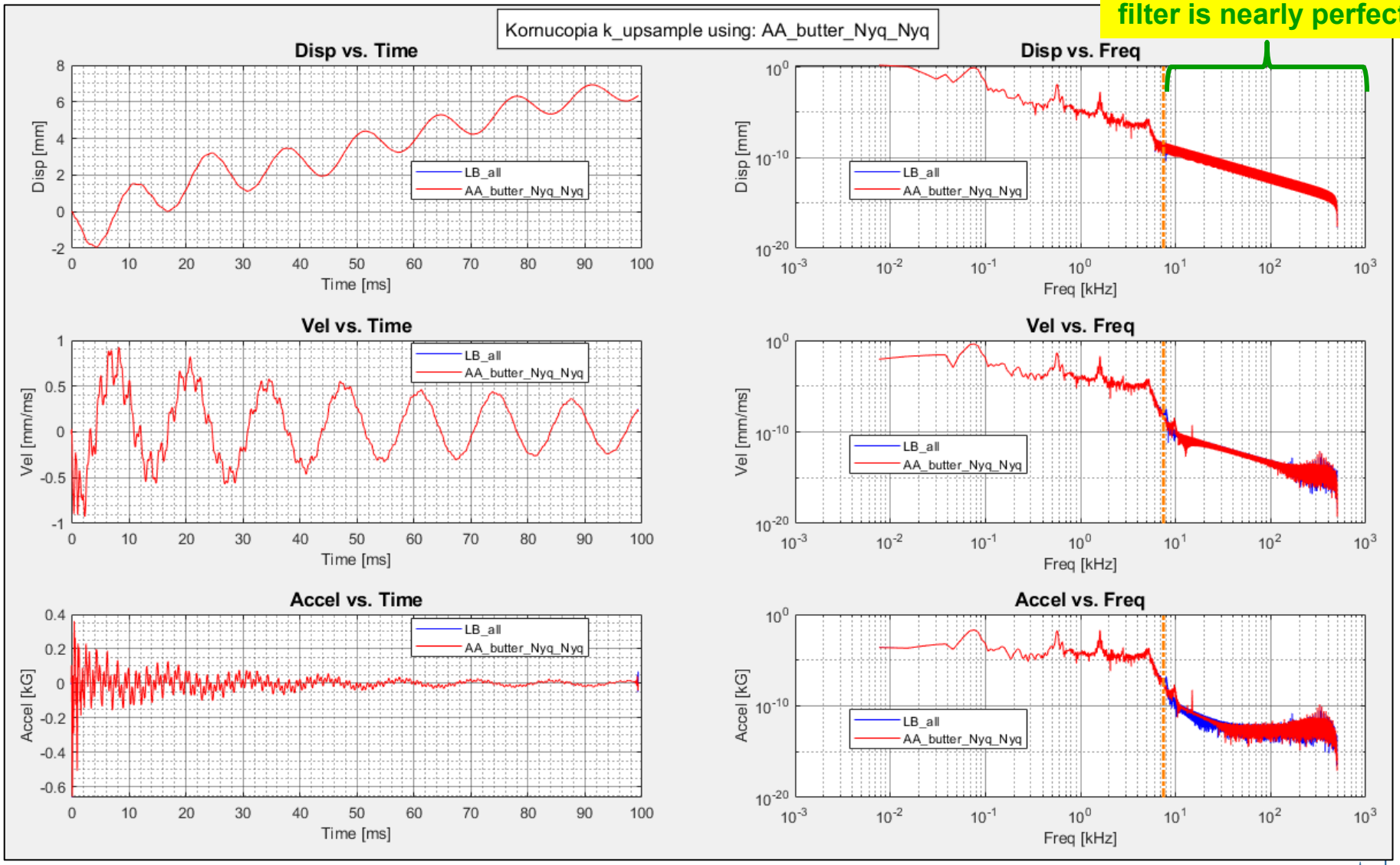
# Classic Zero-Fill and LP Filter Upsampling via Kornucopia's Prior Algorithm within `k_upsample`




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# Classic Zero-Fill and LP Filter Upsampling via Kornucopia's NEW Algorithm within `k_upsample`

**Attenuation by the LP filter is nearly perfect**

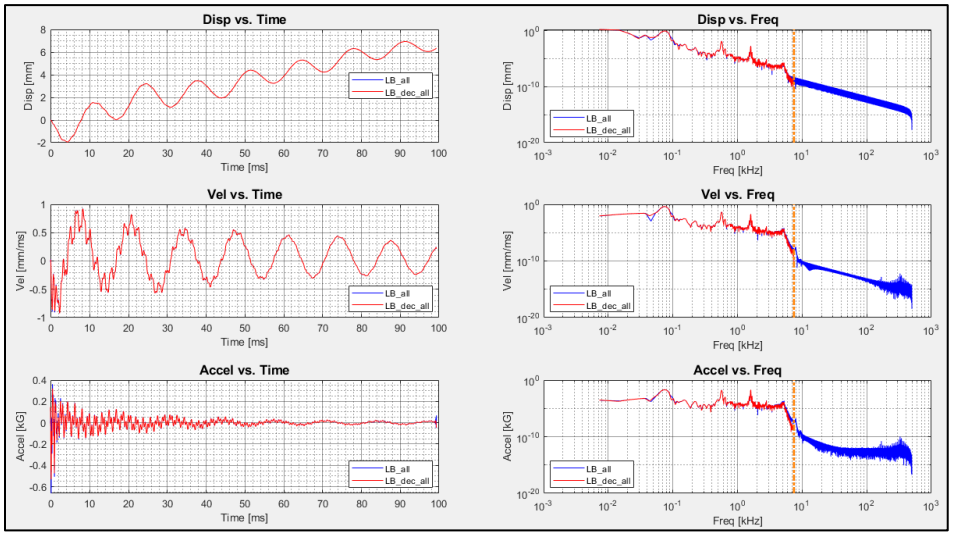



**Upsample looks excellent in ALL forms**

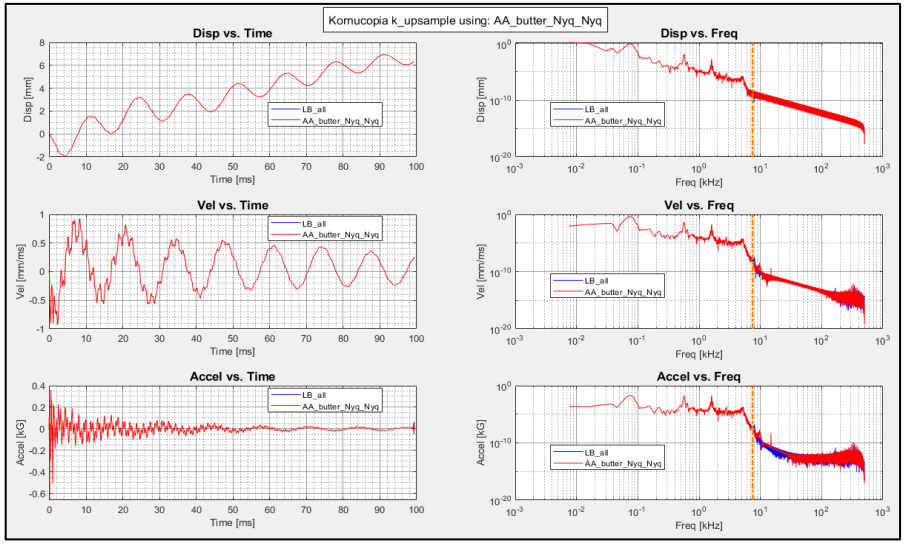
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# Classic Zero-Fill and LP Filter Upsampling via Kornucopia's NEW Algorithm within `k_upsample`

**Where we started:**
**Original signal and its Decimated form**




**Where we ended:**
**Original signal and Upsampled version of Decimated form**




**Upsample looks excellent in ALL forms**

- **Keys to improvement:**
  1. Focus on ensuring false content above fNyq_dec is attenuated.
  2. Much stronger LP filter based on a 10th-order AA filter.
  3. Configurable additional LP cleanup filter (default = Nyq) further attenuates remaining out-of-band content.

Available in
***Kornucopia® ML™***
***Version 5.3-05***

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