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Estimating Stress from a PVSS

Excerpts from the course

Understanding Shock Spectra and PVSS Analysis

Instructor: Ted Diehl, PhD

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Estimating Stress from PVSS

This document is a condensed excerpt from our course
Understanding Shock Spectra and PVSS Analysis

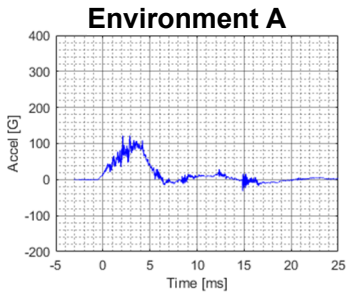
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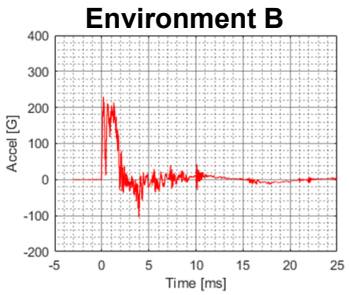
1. **Estimating Shock Severity and Payload Stresses from Shock Response Analyses**
2. **Demonstration of Using Hunt-Gaberson Equation on the Modified Scavuzzo Problem And Why It Cannot Be Easily Utilized On the More Complicated Plate Shock Study**

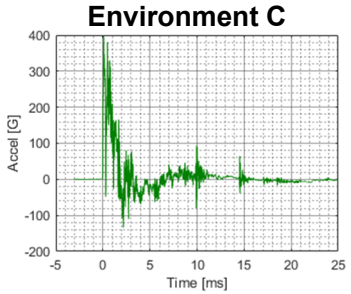
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Estimating Shock Severity and Payload Stresses from Shock Response Analyses

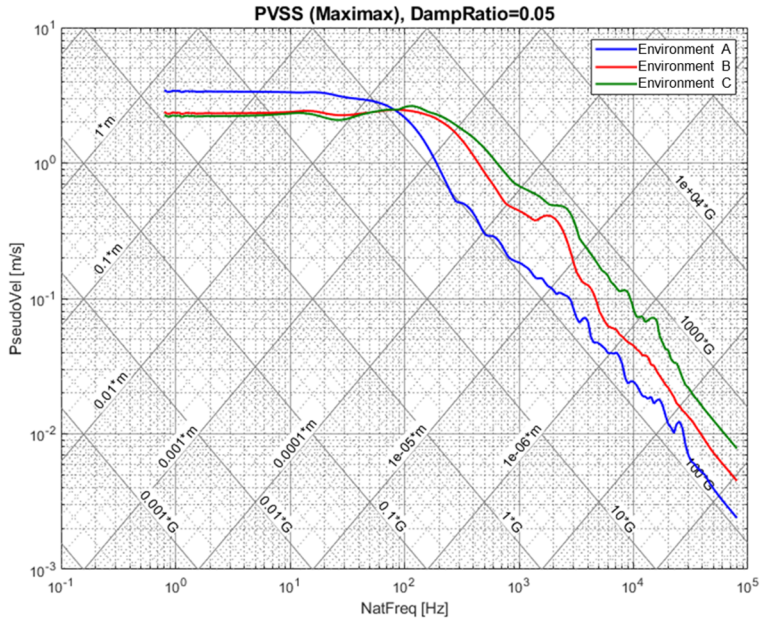






Which environment is worse?

What type of estimates of payload stress can be made?

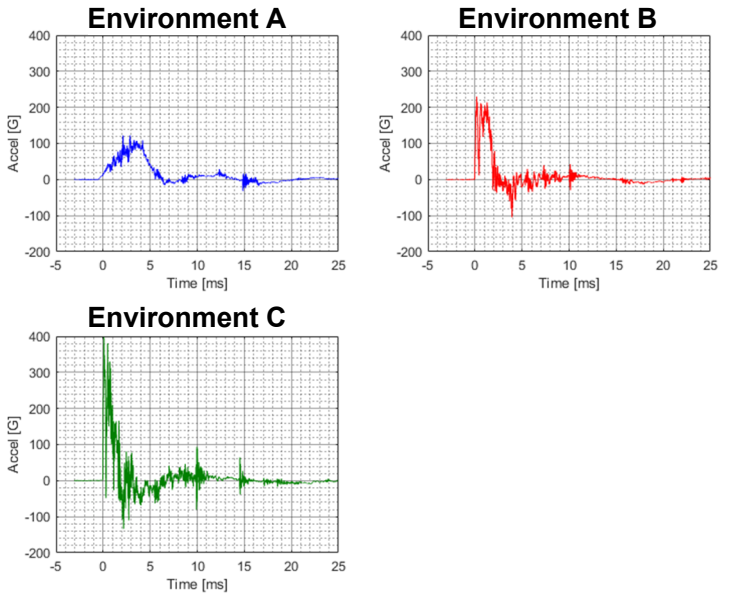


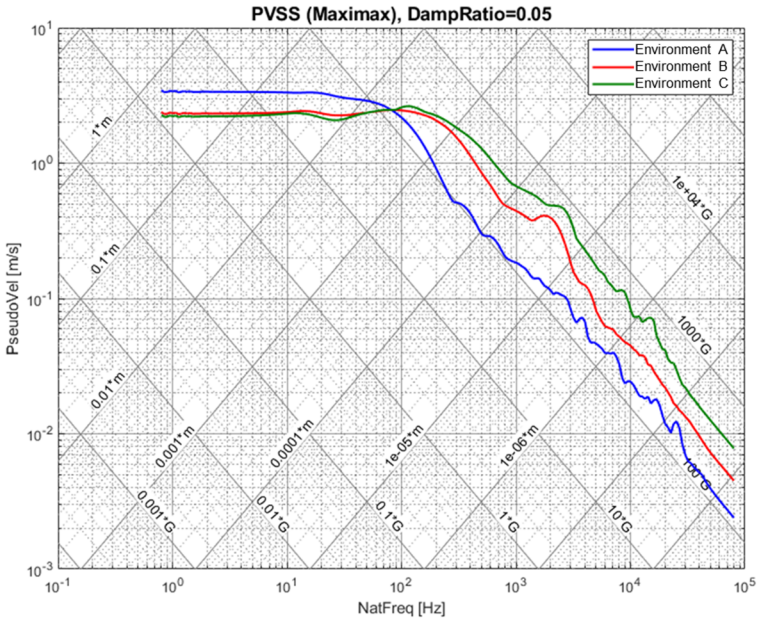
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Estimating Shock Severity & Payload Stress

Listed below in order of sophistication are methods to estimate a given shock environment's severity and, in some cases, estimate a payload's maximum stress due to the shock. Comments on their validity are provided on the following slides.

1. **Peak absolute acceleration in the time domain**
2. **Largest total change in velocity, ΔV (time domain)**
3. **Using Pseudo-Velocity (V_p) plateau from a PVSS**
4. **Using the V_p value from a PVSS at the dominant natural frequency of the payload**
5. **Obtaining a scaled G-load from PVSS at dominant natural frequency of payload for a static gravity-load analysis**
6. **Performing an SRS-driven modal analysis**
7. **Performing a nonlinear transient simulation driven by the full transient shock input**





Only option #7 can include nonlinear payload response effects.

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1) Peak Absolute Acceleration in the Time Domain

- **Approach**: Take the peak absolute acceleration value from the time domain measurement of the environment and use it as the primary characterization of the shock severity and apply its value in a static G-load stress analysis on your payload.
- **Pros:** None
- **Cons:** Several
  - A terrible idea for characterizing the shock environment or predicting payload stress. Using the peak acceleration is a highly misleading metric for most shock environments.
  - Without including frequency perspective of both the acceleration signal representing the environment and the payload's natural frequencies, attempting to use just the time domain's peak absolute acceleration will lead to a useless characterization of the environment and stress predictions of the payload could be way off, generally excessively high.

We will show with several examples in this course that this is just a terrible approach – don't do it!

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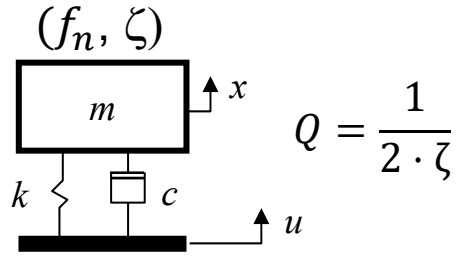
2) Largest Total Change in Velocity, ΔV (Time Domain)

- **Approach**: Integrate the acceleration vs time signal of the environment to yield velocity vs time. Then look for either the largest absolute velocity or the total change in the velocity - so called ΔV.
  - This metric is often cited by those working with vibration testing.
- **Pros:** The concept is rooted in the idea of energy/momentum transfer which is a global property.
- **Cons:** Several
  - Peak Stress is a local, time-dependent property driven by how fast that ΔV is delivered (the slope of the velocity curve, which is acceleration).
  - Consider the following two scenarios related to a passenger (the payload) in a car traveling at 60*mph:
    a) They crash into a solid cement wall, stopping the car in milliseconds. The passenger is likely severely injured or dead.
    b) They apply the brakes and slow to a stop over several seconds. The passenger is not injured in any way.

    *Both events had -60*mph ΔV, but their damage potentials were very different!*
- **Using a global ΔV from the environment is not sufficient to characterize it nor predict payload stresses from it.**

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Remember From Our Closer Look at Acceleration SRS & PVSS



$$Q = \frac{1}{2 \cdot \zeta}$$

- Traditional Absolute Acceleration transfer function:

$$H(s)_{AbsAccel} = H(s)_{Accel} = \frac{\ddot{x}(s)}{\ddot{u}(s)} = \frac{\frac{\omega_n \cdot s}{Q} + {\omega_n}^2}{s^2 + \frac{\omega_n \cdot s}{Q} + {\omega_n}^2}$$

- Transfer functions based on relative displacement:

$$H(s)_{RelDisp} = \frac{x(s) - u(s)}{\ddot{u}(s)} = \frac{-1}{s^2 + \frac{\omega_n \cdot s}{Q} + {\omega_n}^2}$$

$$H(s)_{PVSS} = \frac{[x(s) - u(s)] \cdot \omega_n}{\ddot{u}(s)} = \frac{-\omega_n}{s^2 + \frac{\omega_n \cdot s}{Q} + {\omega_n}^2}$$

$$H(s)_{EqStaticAccel} = \frac{[x(s) - u(s)] \cdot {\omega_n}^2}{\ddot{u}(s)} = \frac{-{\omega_n}^2}{s^2 + \frac{\omega_n \cdot s}{Q} + {\omega_n}^2}$$

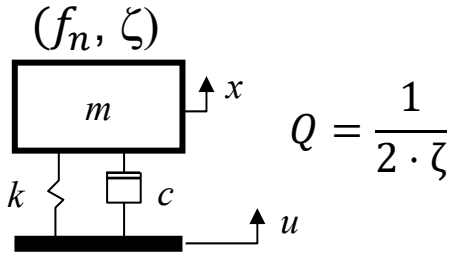
Note: $V_p = (x - u) \cdot \omega_n$ and $A_p = (x - u) \cdot \omega_n^2$

Key observations

- In general, we exactly map between **RelDisp**, **PseudoVel** and **EqStaticAccel** via the multiplication or division by ω_n or ω_n^2, regardless of the damping value.
- If damping ratio, ζ, is zero, then $Q = \infty$ and we see the traditional $H(s)_{AbsAccel}$ is identical to $H(s)_{EqStaticAccel}$.
  - If damping is small, the mapping of traditional AbsAccel to EqStaticAccel is approximate.

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A Comment About Phase and Pseudo Velocity (V_p) and Pseudo Acceleration (A_p)



- Traditional Absolute Acceleration transfer function:

$$H(s)_{AbsAccel} = H(s)_{Accel} = \frac{\ddot{x}(s)}{\ddot{u}(s)}$$

- Transfer functions based on relative displacement:

$$H(s)_{RelDisp} = \frac{x(s) - u(s)}{\ddot{u}(s)}$$

$$H(s)_{PVSS} = \frac{[x(s) - u(s)] \cdot \omega_n}{\ddot{u}(s)}$$

$$H(s)_{EqStaticAccel} = \frac{[x(s) - u(s)] \cdot {\omega_n}^2}{\ddot{u}(s)}$$

Note: $V_p = (x - u) \cdot \omega_n$ and $A_p = (x - u) \cdot \omega_n^2$

- The Pseudo Acceleration (A_p) is exactly the **EqStaticAccel**.

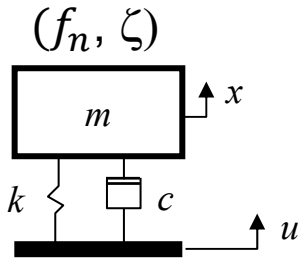
Key points

- These pseudo transfer functions $H(s)_{PVSS}$ and $H(s)_{EqStaticAccel}$ are defined as scalar multiples of ω_n times relative displacement.
  - Unlike physical derivatives, they purposefully ignore the vibration convention of multiplying by terms like $j \cdot \omega_n$ and $-\omega_n^2$. These complex and negative multipliers generate 90° and 180° phase shifts, respectively.
  - By using only positive ω_n scalars, the pseudo quantities stay phase locked with the relative displacement which is the true driver of stress.

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The Fundamental Driver of Stress in a Payload / Structure

- **The Physics**:
  - In any elastic structure, stress (σ) is governed by Hooke's Law:
    $\sigma = E \cdot \epsilon$ where E is Young's modulus and ϵ is strain.
  - Linear 1-D analysis, strain is related to deformation of the payload:
    $\epsilon = (x - u)/L$ where L is a reference length.
  - From our PVSS analysis, Pseudo Velocity is via a scaled relative displacement:
    $V_p = (x - u) \cdot \omega_n$ where the payload natural frequency is $\omega_n = \sqrt{k/m}$
  - If our simple 1-D SDOF system is idealized as a homogeneous rod, we have:
    Stiffness: $k = A \cdot E/L$ and volumetric mass density: $\rho = m/(A \cdot L)$
    where A is the cross-sectional area of the rod.
  - Combining the above relationships and simplifying yields:
    $\sigma = V_p \cdot \sqrt{E \cdot \rho}$ or for other simple structural shapes $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$
    where we have generalized the equation using a nondimensional shape factor, K (note that shape factor K is different from stiffness k).



Input (Driving Motion)

This is the same equation stated by Gaberson [44, 88] & Hunt [89].

- Stress is a function of a structure's relative displacements which is equally stated as stress is a function of Pseudo Velocity!
  - Pseudo-Velocity is a proxy for relative displacement.

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The Fundamental Driver of Stress in a Payload / Structure (Continued)

- From the previous slide we concluded:
  - Stress is a function of a structure's relative displacements.
- **FEA Structural Models**
  - *All FEA structural models compute stresses from the physics stated above.*
  - *This is true for static models, modal dynamic models (steady state or transient), transient dynamic models, SRS-driven models, and even explicit dynamics models (that include wave propagation).*
    - All of these types of analyses ultimately compute stress based on a material model acting on strains, and the strains are derived from structure's relative displacements normalized by reference lengths.
    - An interesting note: explicit dynamics models can capture both lower frequency modal behavior as well as very fast high frequency wave propagation whereas modal models can only represent the former.
  - Note: when computing an SRS-driven modal model, the acceleration shock spectrum supplied as input to drive (load) the model's modes is actually converted internally by the FEA software to a relative displacement spectrum by dividing the acceleration values by the squared natural frequency of each mode. The stress is then computed from the modal displacement shapes, modal participation factors, and the scaled SRS relative displacements.

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Stress From Pseudo-Velocity (V_p) Via the Hunt-Gaberson Eq'n

- As derived and generalized we have:

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

where:

  - K is a dimensionless shape factor for the payload.
  - V_p is the pseudo velocity from a PVSS.
  - E and ρ are the payload's Young's modulus and volumetric mass density.
  - $\sqrt{E \cdot \rho}$ is the material's *characteristic specific impedance*.

- The stress formulation above is intended for first mode calculations only, and for simple homogeneous payloads (beams, plates, etc.).
- For more complicated multi-material payloads, or higher / multiple modes, more advanced approaches are needed.

The table of shape factors, K, to the right is from Irvine, [Ref 90]. The modest list of structures shown has factors ranging from 0.15 to 4.0, although smaller or higher values are possible.

Remember, V_p is just relative payload displacement scaled by the payload's natural frequency.

Table 5.3. K Factors Summary

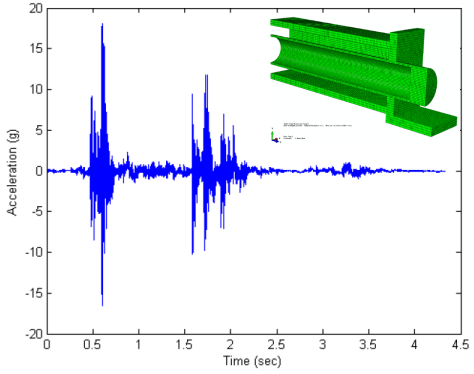
Structure	K
Straight Rod Longitudinal	1
Straight Rod Longitudinal with End Mass	1 to 2.2
Tapered Rod	0.5 to 1
Ring, Breathing Mode	1
Beam Bending	1.0 to 3.8
Cantilever Beam Bending with Applied Static Axial Load (See Note 2)	1.57 to $\sqrt{3}$
Beam Bending with Point Mass	1.1 to 4.0
Beam Bending with End Springs	0.15 to 2.0
Multispan Beam Bending	1.4 to 2.7
Rectangular Plate Bending	1 to 2
Circular Plate Bending	0.87 to 1.0
Overall Range	0.15 to 4.0

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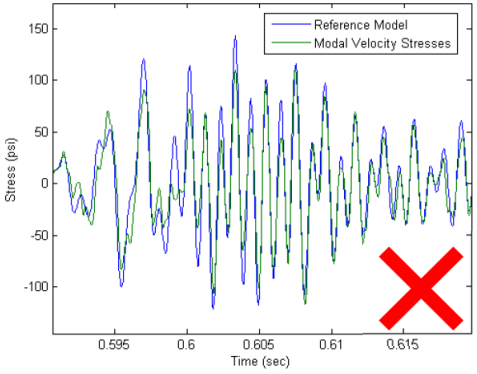
What About "Stress is Proportional to Modal Velocity"?

- Many sources state that stress is proportional to modal velocity.
  - Refs: Bateman & Gaberson [22], Gaberson [44, 88], Hunt [89], and Irvine [90].
    - This comes from analytical analysis of simple rods, beams and similar.
  - Definitions:
    - **Modal Displacement** (q_i): This represents the generalized coordinate or the amplitude of the i^{th} mode shape over time.
    - **Modal Velocity** ($\dot{q}_i$) is the time derivative of the modal displacement (q_i).
  - Manko [91] clearly demonstrates that for complex transients and realistic structures that the actual time-varying modal velocity ($\dot{q}_i$) is not perfectly proportional to stress because of a ¼ period phase lag. He demonstrates that modal stress is physically proportional to modal displacement (q_i), not the derivative ($\dot{q}_i$).

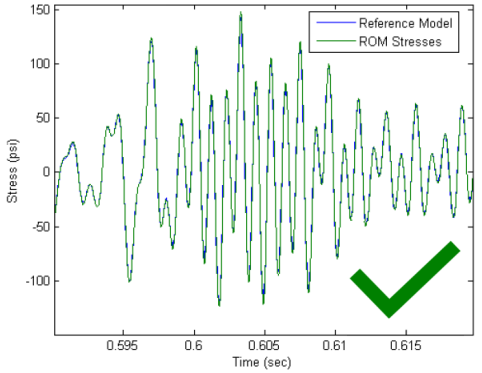
Realistic model driven by transient shock



Modal velocity-based solution does not match benchmark



Modal displacement-based solution matches benchmark



Modal velocity correlates with stress only under specific phase relationships (e.g., harmonic steady-state). For transient shock, this assumption breaks down.

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3) Using Pseudo-Velocity (V_p) Plateau from a PVSS

- **Approach**:
  - Compute a PVSS from the environment's acceleration time-history. Then use the nominal $\boldsymbol{V_p}$ value that represents the plateau region as the characterization of shock severity of the environment.
  - Additionally utilize this $\boldsymbol{V_p}$ value to estimate payload stress per the Hunt-Gaberson equation we previously stated, namely $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$.
  - This type of approach was championed by Dr. Howard Gaberson for assessing shock severity in Naval vessels where the natural frequencies of their payloads tended to be in the region of the PVSS plateau.
- See the next slide for Pros & Cons of this approach.

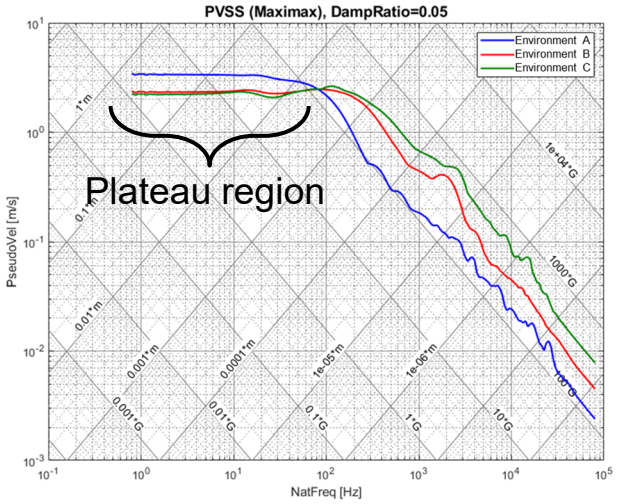
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3) Using Pseudo-Velocity (V_p) Plateau from a PVSS (Continued)

- *Formulation that Stress is Proportional to Pseudo Velocity*

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

- **Pros:** <u>If the payload's primary natural frequency is in the PVSS plateau region:</u>
  - Then this is a valid approach of shock severity characterization. Environments with large plateau $\boldsymbol{V_p}$ values will generate greater shock severity.
  - Then for simple payload structures with known shape factor (K) and material properties, maximum stresses can be estimated based on the 1st mode only.
    - For more complicated payload structures (unknown shape factor (K) and/or multiple materials), or when higher modes are important, the equation potentially provides a general guide of what influences the maximum stress due to the shock.

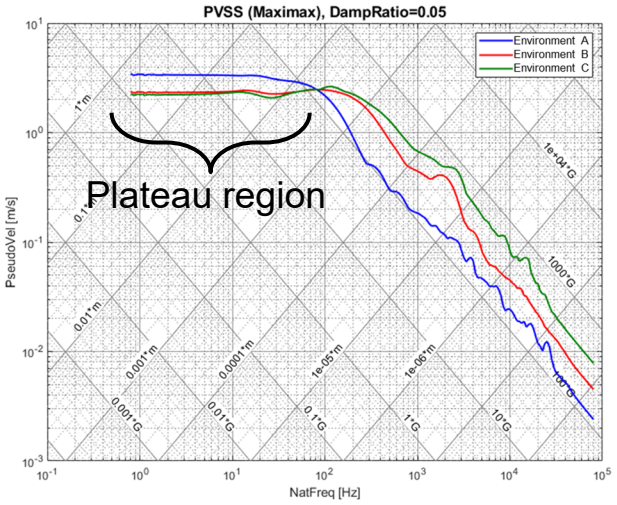


- **Cons:**
  - If the payload's primary natural frequencies are NOT in the PVSS plateau region, then the wrong value of $\boldsymbol{V_p}$ is being utilized to characterize the environment and to estimate payload stress.
    - On the graph on the right, payloads beyond 100*Hz are NOT under the plateau.

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4) Using the V_p Value from a PVSS at the Dominant Natural Frequency of the Payload

- **Approach & Pros**: An improved version of approach #3) "Using the Pseudo-Velocity ($\boldsymbol{V_p}$) Plateau from a PVSS" is created by switching from the plateau region to the $\boldsymbol{V_p}$ value at the first natural frequency of the payload.
  - This approach requires some kind of estimate of the first natural frequency of the payload. Without it, you really cannot claim, in general, which environment would be worse nor what stress levels are likely to develop from the shock environment.
  - The plot of PVSS spectra below clearly shows that Environment A is worst for payloads with natural frequencies in the plateau region, but it is the least damaging for payloads with natural frequencies to the right of the plateau.
- **Cons:**
  - Computing stress requires knowledge of the K shape factor: $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$
  - For payloads with complicated construction, this equation is too simplistic and insufficient.
  - If additional modes will become significantly excited by the shock environment, this approach will not predict the total resultant stresses correctly.
    - This issue is shown later in this course.



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5) Obtain a Scaled G-Load from PVSS at Dominant Natural Frequency of Payload for a Static Gravity-Load Analysis

- **Approach & Pros**: The PVSS plotted on 4-CP (four-coordinate paper) makes it easy to equally assess the spectrum in terms of any of the following:
  - A) Pseudo Velocity, B) Equivalent Static Acceleration, C) Relative Displacement.
    - In general, we exactly map between **RelDisp**, **PseudoVel** and **EqStaticAccel** via the multiplication or division by ω_n or ω_n^2, regardless of the damping value.
  - The 1st mode of most structures is similar in shape to a gravity load. We approximate the 1st mode max stress response via a scaled gravity load static analysis. The PVSS, evaluated at the payload natural frequency provides the G-loading needed.
  - Using FEA, more payload geometry / material complexity can be addressed.
    - Because Pseudo Velocity and Equivalent Static Acceleration (both assessed at the same natural frequency) are one-to-one mappable, this implies they are essentially providing the same loading amplitude. Since a scaled gravity load creates a deformed shape that is typically very similar to a 1st-mode shape (but not exactly the same), this FEA approach should yield similar results to the Hunt-Gaberson eq'n but be more generally capable due to FEA's nature.
- **Cons:**
  - Only a scaled gravity load approximation of the payload's 1st mode deformation is analyzed. The influence of higher modes is completely ignored.

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6) Performing an SRS-Driven Modal Analysis

- **Approach & Pros**: Most FEA software requires an acceleration SRS input. If using a PVSS, the curve must be mapped to acceleration. Theoretically, the EqStaticAccel SRS (A_p) is the correct driver as it's derived from relative displacement. In practice, the AbsAccel SRS is often utilized, which is a valid approximation only for light damping.
  - During the solution, FEA software internally maps the acceleration SRS input to a relative displacement spectrum by dividing the acceleration values by the squared natural frequency of each mode being analyzed. The stress for each mode is then computed from the modal displacement shape, modal participation factor, and the scaled SRS relative displacement for that mode.
    - Because no phase information exists in the driving SRS, there is no theoretically correct method to combine the modal results. Instead, one of a number of approximate modal combination methods are offered including but not limited to **absolute sum**, **SRSS** (Square Root of Sum of Square), and **NRL** (Naval Research Lab) which takes the largest modal response and adds that to the SRSS of the remaining modes.
  - Because multiple modes are included, this approach is generally superior for stress estimation as compared to the prior approaches based on $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$.
- **Cons:**
  - Modal summation (and directional summation) are approximations.
  - No nonlinearities can be included.

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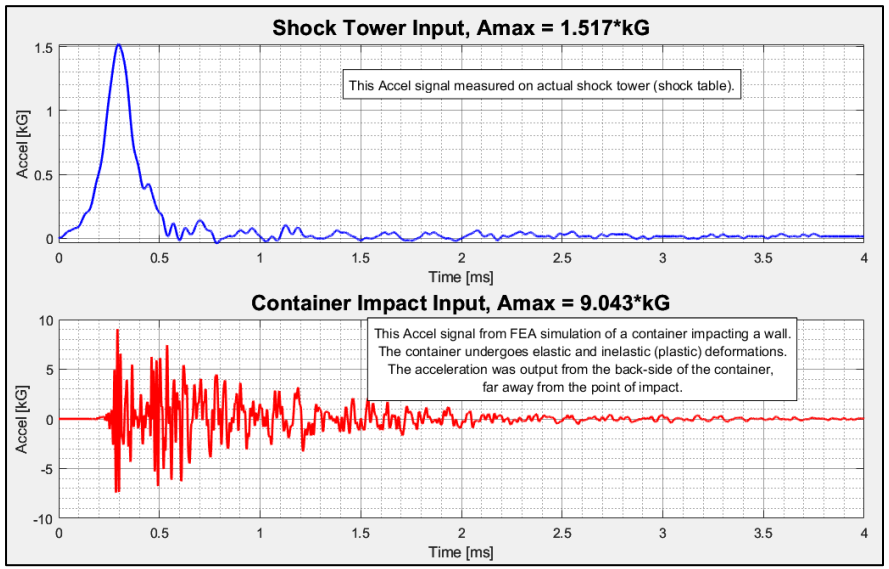
7) Performing a Nonlinear Transient Simulation Driven by the Full Transient Shock Input

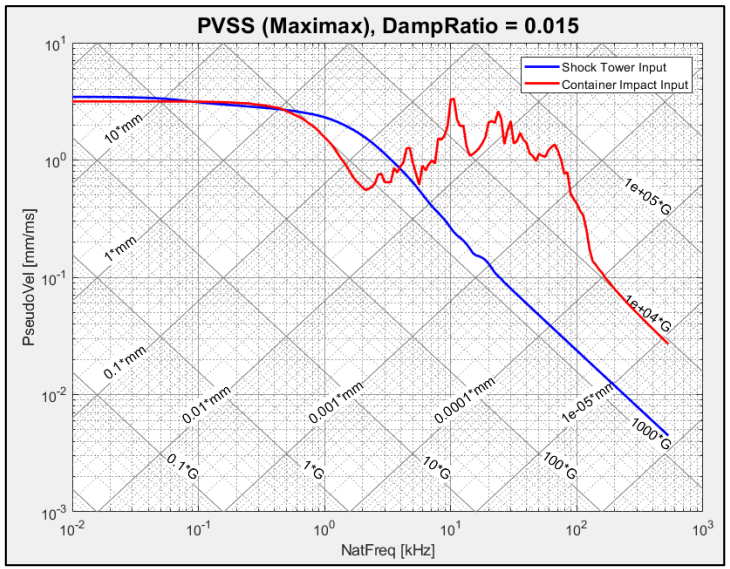
- **Approach & Pros**: This simulation is typically done with FEA, either using implicit or explicit dynamics via software from Abaqus, Ansys, LS-Dyna, or similar. The models are typically fairly detailed and capable of representing payloads that range from simple to rather complicated, including payloads with multiple materials, contact behavior, and other nonlinear characteristics.
  - This approach is the most sophisticated, but also the most difficult to perform as the analyst must be skilled on **advanced FEA** and **applied DSP** to perform the analysis properly and interpret the results, including potentially applying filtering to remove solution-based noise in some of the computed quantities.
- **Cons:**
  - A skilled analysis must perform the analysis, otherwise you are just computing cyber-dope! (garbage-in/garbage-out results)
  - Models can be very large and require significant computing resources and compute time.

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Complex PVSS Spectra Require Additional Care/Thought

- Consider the two shock environments presented below.
  - The Shock-Tower delivers a pulse-like shock with a fairly classical PVSS curve shape (Plateau followed by “the knee” and a constant acceleration asymptote).
  - The Container Impact shock is more complicated. Its PVSS curve deviates sharply from the classic response beyond the “knee”. Payloads with important modes in this region (>1*kHz) may find their higher modes contribute significantly, and maybe even become the dominant stress generators from such a shock.
    - The Hunt-Gaberson equation, $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$, will not be predictive in this scenario, but SRS driven modal models and full transient FEA simulations can be.

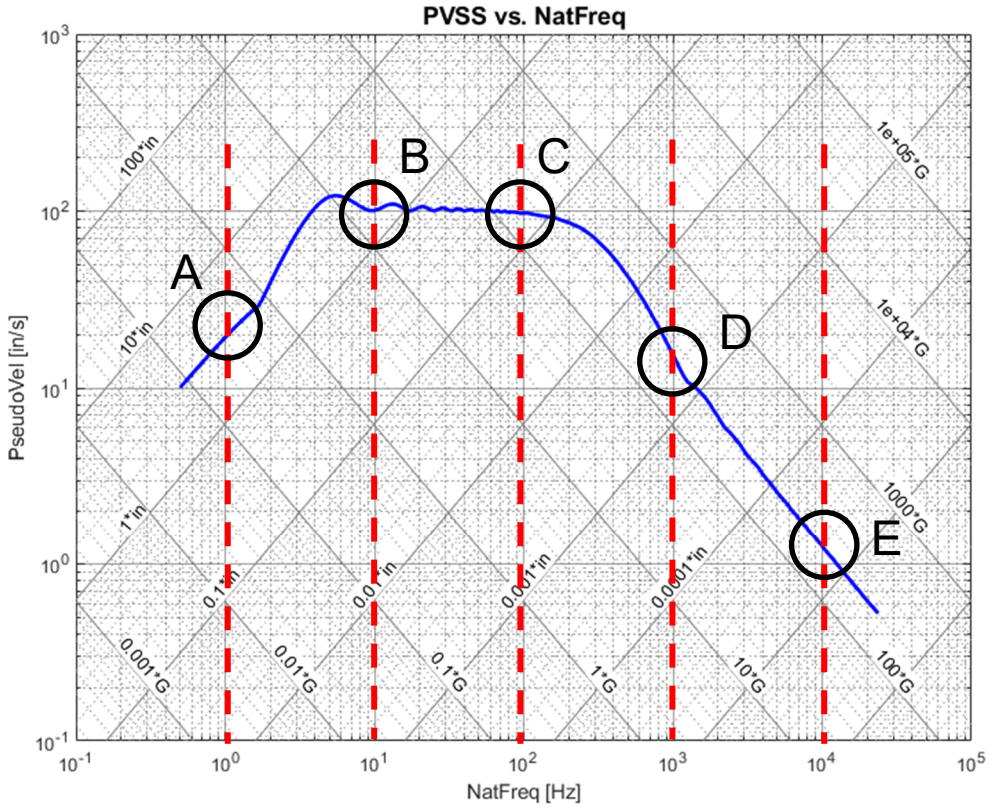




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How Does Stress Change as a Function of fn_payload?

- Consider a classic PVSS curve that contains all three guideline regions:
  1. Constant relative displacement
  2. Plateau (nearly constant V_p)
  3. Constant equivalent static acceleration, A_p.



- We denote 5 locations on the graph (A – E) where the 1st natural frequency of a *designable payload* intersect the PVSS curve as the payload changes its natural frequency from 1*Hz to 10*kHz.
- The Hunt-Gaberson equation states:

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

- To answer our question, we need to create a designable payload for which we can vary its natural frequency significantly.
  - For any structure, the natural frequency is a function of material properties, geometry, and boundary conditions. Irvine [90] clearly shows that the K shape factor is also a function of geometry and boundary conditions.

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How Does Stress Change as a Function of fn_payload? (continued)

- **Challenge:** For our notional study, we desire to assess the Hunt-Gaberson equation for a "designable payload" capable of changing its natural frequency over 4 orders of magnitude! This is no trivial task.

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

$$\omega_n = f(E, \rho, \text{geom.}, \text{BCs})$$

- Added complexities to the task are:
  - For real materials, E and ρ are not completely independent (i.e. you cannot hold ρ a constant and magically vary E by orders of magnitude).
  - The stress formula shows that if we exclude the shape factor K that the stress is not a function of geometry, but only just material properties E and ρ.
  - From Irvine [90] and others, the shape factor K is a function of geometry and BCs (boundary conditions), albeit via complicated analyses that are summarized in lookup tables which are not formulated as a function of natural frequency directly.
    - For a rod in axial deformation, the shape factor $K = 1$ and is independent of the rod's length or cross-sectional area (from Refs [22 & 90]).
    - For a solid rectangular beam in cantilever bending, the shape factor is $K = \sqrt{3} \approx 1.732$. With clamps on both ends, $K \approx 2.18$ (both from [90]). These cases are limited to Bernoulli-Euler beam theory. (Note: At extremely short lengths (high NatFreqs), Timoshenko effects would cause K to deviate).
- What is this telling us ???

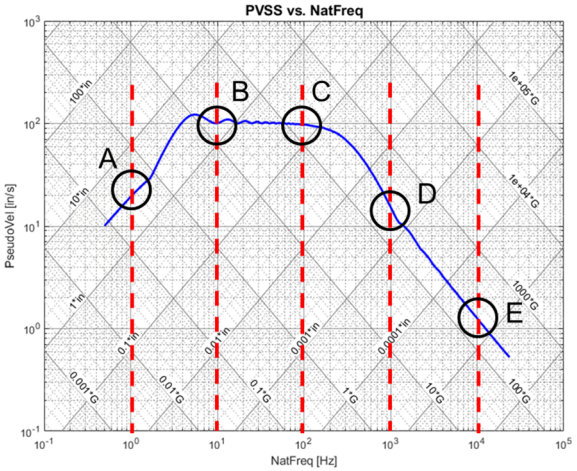
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How Stress Changes as a Function of fn_payload

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

$$\omega_n = f(E, \rho, \text{geom.}, \text{BCs})$$

1. For a given geometry type (i.e. a rod in axial deformation or a rectangular beam with one or two cantilever clamps), the shape factor K is a constant that is not dependent on changes in geometry such as thickness or cross-sectional area.
   - So, while natural frequency will change when thickness or cross-sectional area is changed in such a payload, the stress will not change unless such a natural frequency change causes V_p to change (as predicted by the PVSS).



2. For a homogeneous payload, once a structural shape and BCs are chosen, the material properties E and ρ become the only way to significantly influence the stress, and that effect is mitigated to some degree by a square root operator.

3. If the payload is not homogeneous, like an axial rod with a lumped mass on the end or a cantilever beam with a lumped mass located on its free end or elsewhere, then the shape factor K becomes a function of the ratio of the lumped mass relative to the mass of the rod (or beam) (Irvine [90, Tables 10.1 and 16.1]), with K increasing and natural frequency decreasing as the mass ratio increases. In this scenario, both the payload's natural frequency and shape factor K are thus a function of this mass ratio and then the stress has some additional dependency on payload natural frequency.

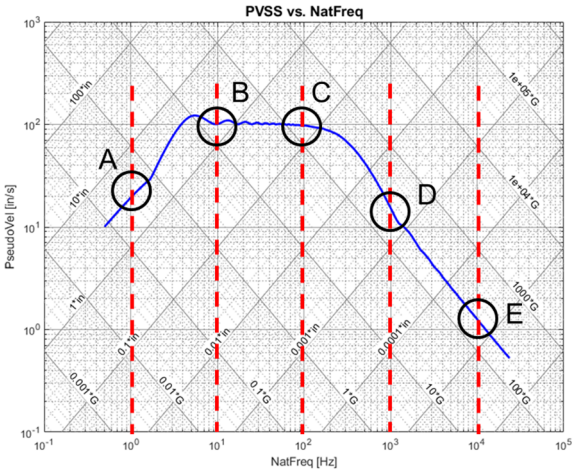
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How Stress Changes as a Function of fn_payload

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

$$\omega_n = f(E, \rho, \text{geom.}, \text{BCs})$$

4. For cases with lumped masses in the payload, since K increases and natural frequency decreases as the lumped mass increases, you need to account for the changing K and be aware of where your payload is on the PVSS curve to ensure you utilize the correct V_p value.
   - Depending on where on the PVSS curve the payload designed started (with no lumped mass), as you increase the lumped mass value, you may find that the PVSS curves show an increasing V_p, or V_p staying relatively constant, or a decreasing V_p.
     This will of course influence the stress predicted.



Irvine's K -factor tables [90]

Table 10.1. Rod with End Mass, K Values

Rod Mass (lbm)	End Mass (lbm)	Ratio of End Mass to Rod Mass	Fundamental Frequency (Hz)	K
7.54	3.77	0.5	1406	1.13
7.54	7.54	1	1122	1.32
7.54	15.1	2	851	1.65
7.54	22.6	3	714	1.92
7.54	30.2	4	625	2.17

Table 16.1. Cantilever Beam with End Mass, K Values

Rod Mass (lbm)	End Mass (lbm)	Ratio of End Mass to Beam Mass	Fundamental Frequency (Hz)	K
0.110	0	0	103.1	2.0
0.110	0.0275	0.25	72.7	2.6
0.110	0.055	0.5	59.2	3.1
0.110	0.0825	0.75	51.2	3.6
0.110	0.110	1	45.7	4.0

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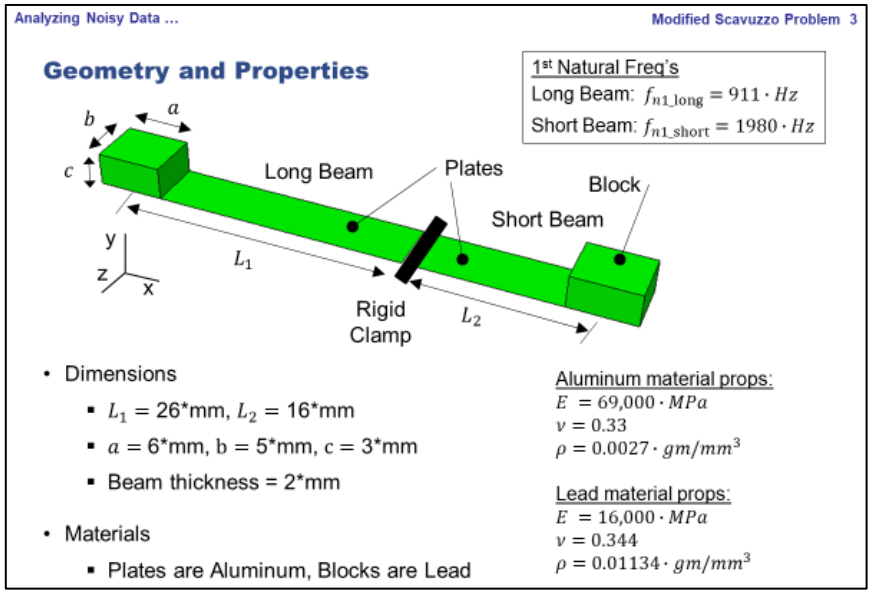
Summary of 7 Approaches for Estimating Shock Severity & Payload Stress

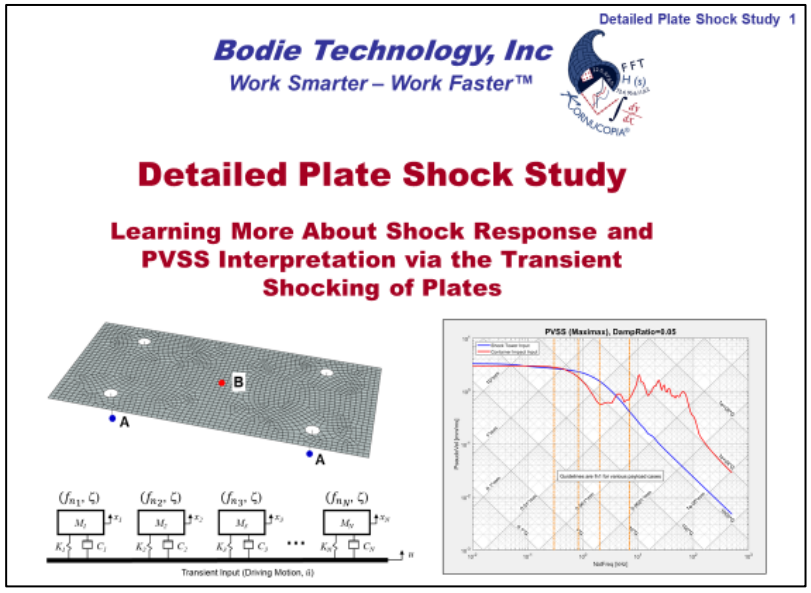
1. **Peak Accel (Time Domain)** – **BAD IDEA, DON'T DO IT.**
2. **ΔV (Time Domain) –** Better than Peak Accel but ignores signal's frequency content and payload's natural frequencies. – **Generally, not suggested**.
3. **V_p in Plateau (PVSS) with Hunt-Gaberson stress formulation** – Better than options 1 & 2 but only valid if payload dominant modes are also in the plateau region. Also, only considers 1st mode and only applicable to simple payloads where you know K-factor.
4. **V_p at f_n of Payload (PVSS) with Hunt-Gaberson stress formulation** – Better than options 1 - 3 because you are looking at the right location (f_n of Payload) but only considers 1st mode and only applicable to simple payloads where you know K-factor.
5. **Scaled Static G-Load (PVSS or Abs Accel SRS)** – Similar to or better than option 4 because when utilized with FEA you can represent a more complicated payload, but only approximates 1st mode participation.
6. **SRS-Driven Modal Analysis** – Better than options 1 – 5 because you include more than just the 1st mode, but combining the modes requires choice of an approximate method because SRS has no phase information. Restricted to Linear Analysis.
7. **Nonlinear Transient FEA Using Transient Shock Input –** Potentially the best stress predictions, but high computational cost plus it requires advanced DSP and FEA skills.

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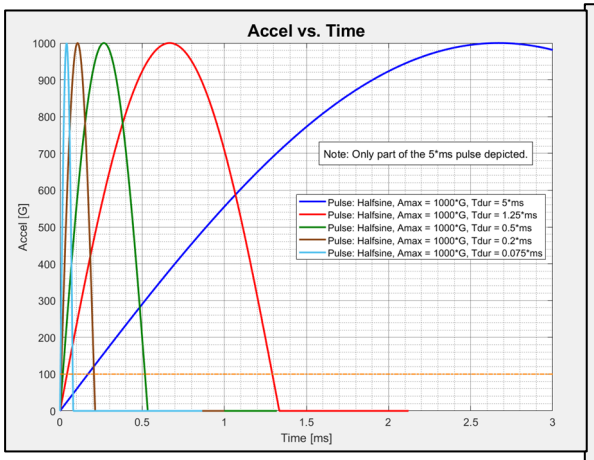
Demonstration of Using the Hunt-Gaberson Equation on the Modified Scavuzzo Problem And Why It Cannot Be Easily Utilized On the More Complicated Plate Shock Study

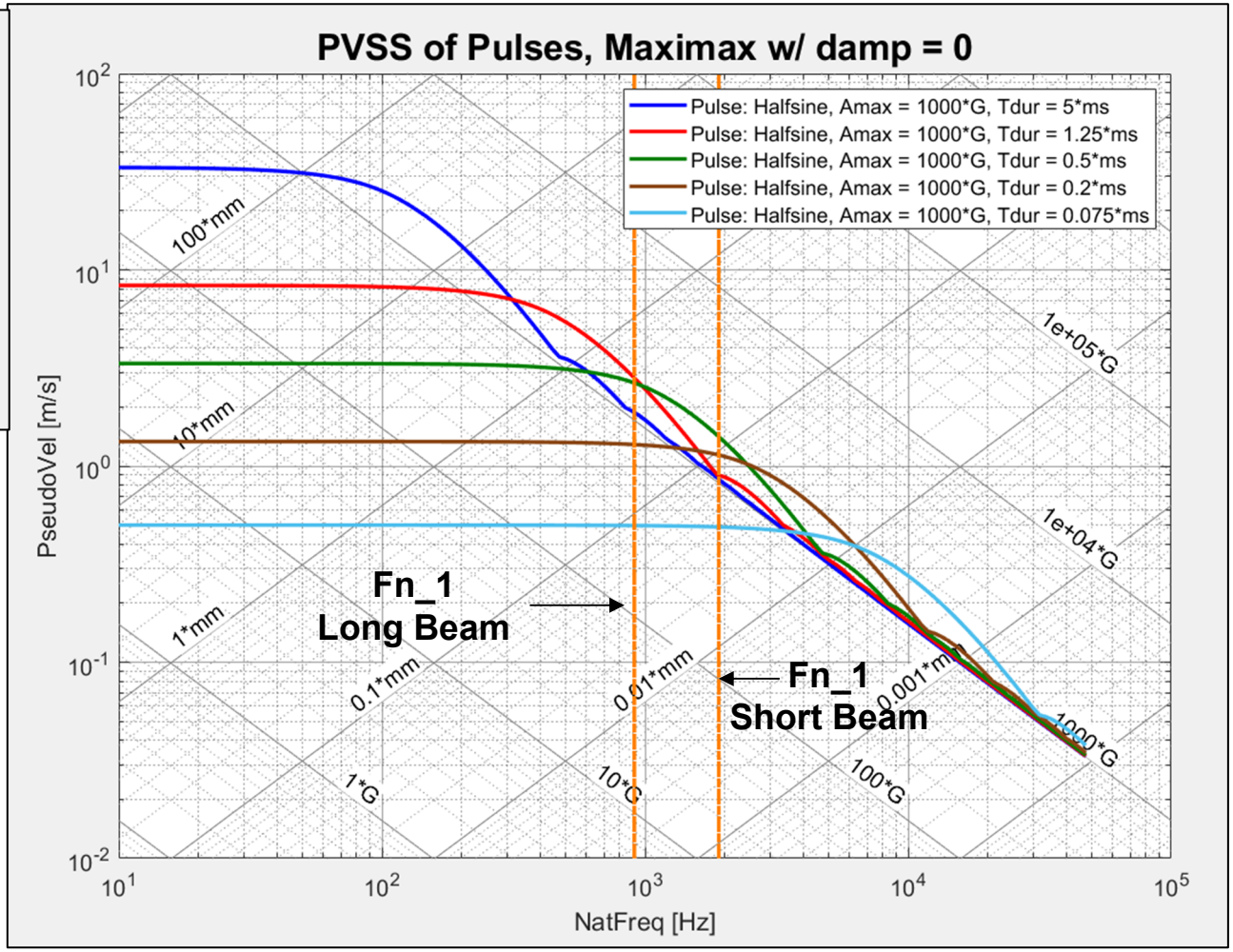




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Soln 3) Utilizing PVSS of Input Accel Along With Natural Frequency Information to Better Understand Shock Behavior





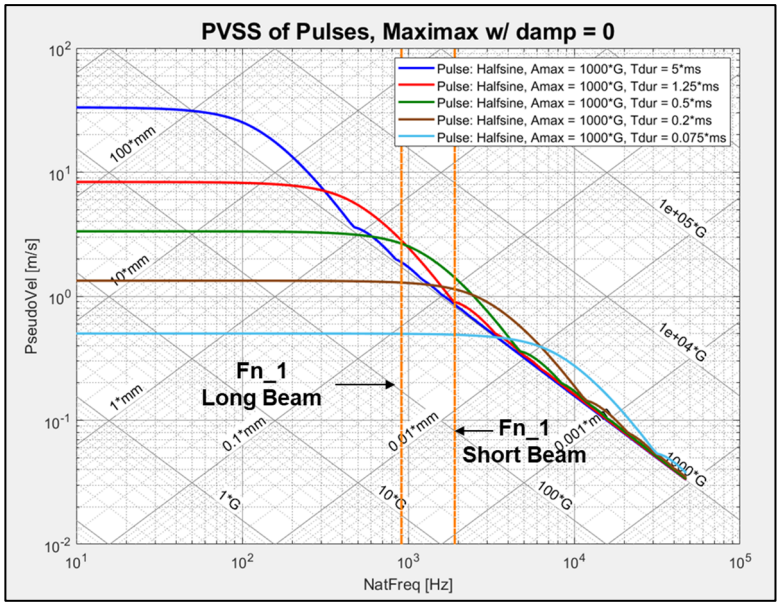
- PVSS shows how SDOF systems would respond to various transient inputs.
- Damping was 0 in this analysis to best mimic the assumption in the Explicit FEA models.
- V_p value needed is NOT always the plateau value!

We obtain the needed V_p and A_p values at the Fn_1 locations for the two analysis techniques we will be using.

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Soln 3a) Apply V_p Value from PVSS @ fn_payload to Hunt-Gaberson Equation

- Hunt-Gaberson Eq'n: $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$
- The table below is derived by finding where the 1st natural freqs of the given beam intersects the PVSS curve for a given pulse.
  - Easily done via `k_interpFunc` from Kornucopia® ML™.
- Assessment of the interpolation yields the needed Pseudo Velocity (V_p) values.




```
SDOF Vp Response
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz
=========================
```

	Short Beam [m/s]	Long Beam [m/s]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	0.8563	1.879	2.190
Halfsine_Amax1000G_Tdur1p250ms	0.8921	2.812	3.150
Halfsine_Amax1000G_Tdur0p5000ms	1.412	2.654	1.880
Halfsine_Amax1000G_Tdur0p2000ms	1.138	1.287	1.130
Halfsine_Amax1000G_Tdur0p07500ms	0.4891	0.4974	1.020

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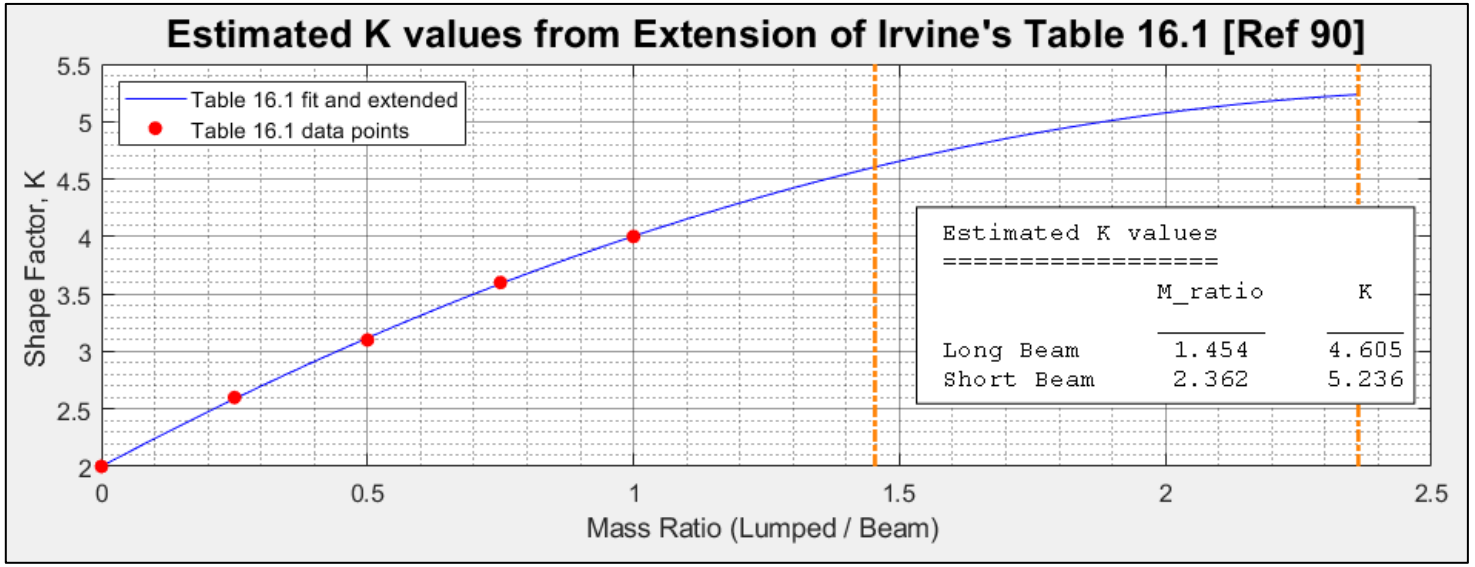
Soln 3a) Determining Shape Factor for Hunt-Gaberson Eq'n

- Hunt-Gaberson Eq'n: $\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$
- The table to the right from Irvine provides guidance on how K varies as a function of the mass ratio.
- The graph below utilized a 2nd-order constrained polynomial to fit Irvine's data and then extend the estimate to the larger mass ratios needed for our problem.

From Irvine [Ref 90]

Table 16.1. Cantilever Beam with End Mass, K Values

Rod Mass (lbm)	End Mass (lbm)	Ratio of End Mass to Beam Mass	Fundamental Frequency (Hz)	K
0.110	0	0	103.1	2.0
0.110	0.0275	0.25	72.7	2.6
0.110	0.055	0.5	59.2	3.1
0.110	0.0825	0.75	51.2	3.6
0.110	0.110	1	45.7	4.0



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Soln 3a) Comparing Full Transient Explicit FEA Results and Estimates from PVSS Analysis Via Hunt Gaberson Eq'n

Max Mises Stress From Transient Soln

	Short Beam [MPa]	Long Beam [MPa]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	62.76	119.1	1.900
Halfsine_Amax1000G_Tdur1p250ms	65.43	176.0	2.690
Halfsine_Amax1000G_Tdur0p5000ms	101.7	161.6	1.590
Halfsine_Amax1000G_Tdur0p2000ms	80.34	79.78	0.9900
Halfsine_Amax1000G_Tdur0p07500ms	36.08	36.15	1.000

Max Mises Stress based on PVSS Vp Response & Hunt-Gaberson Eqn
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz

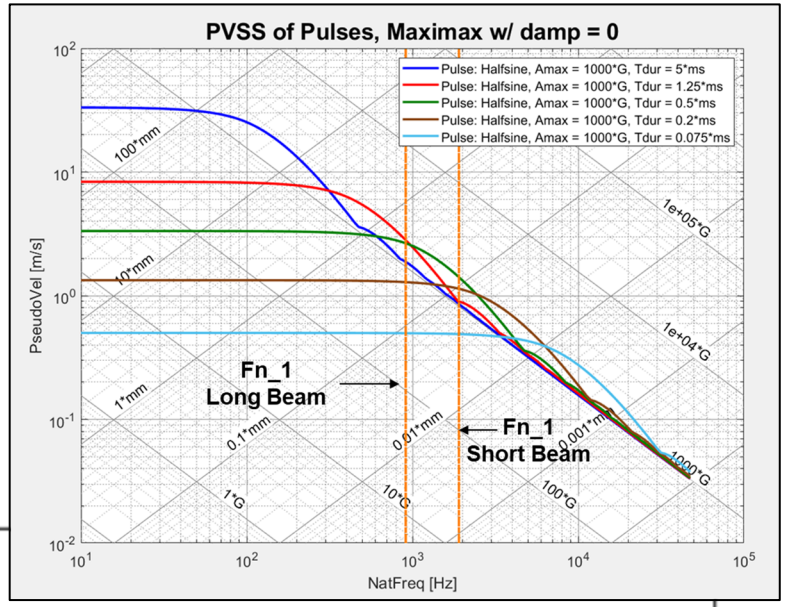
	Short Beam [MPa]	Long Beam [MPa]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	61.20	118.1	1.930
Halfsine_Amax1000G_Tdur1p250ms	63.75	176.7	2.770
Halfsine_Amax1000G_Tdur0p5000ms	100.9	166.8	1.650
Halfsine_Amax1000G_Tdur0p2000ms	81.32	80.89	0.9900
Halfsine_Amax1000G_Tdur0p07500ms	34.95	31.27	0.8900

Very good to excellent correlation for this particular problem

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Soln 3b) Apply A_p Value from PVSS @ fn_payload to For a Scaled Static Gravity Load Analysis

- The table below is derived by finding where the 1st natural frequency of the given beam intersects the PVSS curve for a given pulse.
  - Easily done via `k_interpFunc` from Kornucopia® ML™.
- These are the equivalent static accelerations experienced by the SDOF system.




```
SDOF EqStaticAccel Response
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz
============================
```

	Short Beam [G]	Long Beam [G]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	1047	1097	1.050
Halfsine_Amax1000G_Tdur1p250ms	1091	1641	1.500
Halfsine_Amax1000G_Tdur0p5000ms	1725	1549	0.9000
Halfsine_Amax1000G_Tdur0p2000ms	1391	751.2	0.5400
Halfsine_Amax1000G_Tdur0p07500ms	597.8	290.3	0.4900

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Soln 3b) Estimating Max Mises Stress via A_p from PVSS

SDOF EqStaticAccel Response
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz

2

	Short Beam [G]	Long Beam [G]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	1047	1097	1.050
Halfsine_Amax1000G_Tdur1p250ms	1091	1641	1.500
Halfsine_Amax1000G_Tdur0p5000ms	1725	1549	0.9000
Halfsine_Amax1000G_Tdur0p2000ms	1391	751.2	0.540
Halfsine_Amax1000G_Tdur0p07500ms	597.8	290.3	0.490

From previous linear static gravity load

1

Linear Static Max Stress

Short Beam [MPa/G]	Long Beam [MPa/G]
0.060132	0.10927

Max Mises Stress based on PVSS EqStaticAccel Response
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz

3

	Short Beam [MPa]	Long Beam [MPa]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	62.94	119.8	1.900
Halfsine_Amax1000G_Tdur1p250ms	65.58	179.3	2.730
Halfsine_Amax1000G_Tdur0p5000ms	103.8	169.2	1.630
Halfsine_Amax1000G_Tdur0p2000ms	83.64	82.08	0.9800
Halfsine_Amax1000G_Tdur0p07500ms	35.95	31.72	0.8800

Scale the static stress values above with SDOF EqStaticAccel response from PVSS to estimate dynamic stress.

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Soln 3b) Comparing Full Transient Explicit FEA Results and Estimates from PVSS A_p Analysis

Max Mises Stress From Transient Soln

	Short Beam [MPa]	Long Beam [MPa]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	62.76	119.1	1.900
Halfsine_Amax1000G_Tdur1p250ms	65.43	176.0	2.690
Halfsine_Amax1000G_Tdur0p5000ms	101.7	161.6	1.590
Halfsine_Amax1000G_Tdur0p2000ms	80.34	79.78	0.9900
Halfsine_Amax1000G_Tdur0p07500ms	36.08	36.15	1.000

Max Mises Stress based on PVSS EqStaticAccel Response
fn_1 Short Beam = 1908*Hz
fn_1 Long Beam = 911.0*Hz

	Short Beam [MPa]	Long Beam [MPa]	Long / Short
Halfsine_Amax1000G_Tdur5p000ms	62.94	119.8	1.900
Halfsine_Amax1000G_Tdur1p250ms	65.58	179.3	2.730
Halfsine_Amax1000G_Tdur0p5000ms	103.8	169.2	1.630
Halfsine_Amax1000G_Tdur0p2000ms	83.64	82.08	0.9800
Halfsine_Amax1000G_Tdur0p07500ms	35.95	31.72	0.8800

Very good correlation for this particular problem

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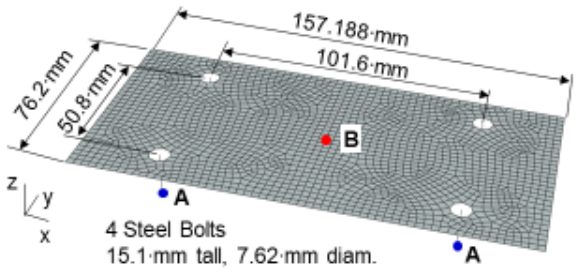
Why the Hunt-Gaberson Eq'n is Not Explored in the Aluminum Plate Shock Study Depicted Below

- To use the Hunt-Gaberson Eq'n we need to know the shape factor, K, for the aluminum plate supported by four steel bolts.

$$\sigma = K \cdot V_p \cdot \sqrt{E \cdot \rho}$$

- The problem — this is a much more complicated payload than a simple rod, beam, or plate with idealized edge constraints.

- The aluminum plate is being used as a simplistic multi-mode payload.
  - Location **A**: Application of shock acceleration (interface to larger structure).
    - DOF 1,2,4,5,6 constrained to zero motion. Acceleration applied DOF 3.
  - Location **B**: Sensing location for plate deformation.
  - **B – A** = relative plate motion.



4 Steel Bolts
15.1·mm tall, 7.62·mm diam.

Kinematic couplings (DOF 1-6) connecting each bolt top to 7.62·mm diam hole in plate.

Abaqus/Explicit S4R shell model of 1.57·mm thick aluminum plate

Aluminum material props:
$E = 69{,}000 \cdot MPa$
$\nu = 0.33$
$\rho = 0.0027 \cdot gm/mm^3$
plate mass = 50.074*gm

Steel material props:
$E = 180{,}000 \cdot MPa$
$\nu = 0.3$
$\rho = 0.00805 \cdot gm/mm^3$

Steel bolts modeled as rigid beams.
Total mass of 4 bolts = 22.173*gm.

- The boundary conditions on the plate are localized around the 4 holes which creates additional stress concentrations.
- The mass and stiffness of the bolts may couple into the plate's dynamic response.

NONE of these facts would be covered in some published K factor you will find in any of the references we have listed previously or any that you are likely to find on the web.

➢ Attempting to use the Hunt-Gaberson Eq'n on this type of problem is not viable unless you do a bunch of FEA to derive the needed K factor for this specific problem. And if you have FEA at that point, just use it!

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Want to Learn More?

This presentation is a condensed excerpt from our course ***Understanding Shock Spectra and PVSS Analysis***

Listed to the right is the full course outline.

Please use the links below for more information or to sign up for our next class!

Understanding Shock Spectra and PVSS Analysis

Topics

- Shock Response Spectrum (SRS) – What is it?
- Various Forms of SRS
- Additional Details on Accel SRS & PVSS
- Estimating Shock Severity & Payload Stresses from Shock Response Analyses
- Converting Peak Accel and Time Duration Shock Specs into SRS
- Example of Applying SRS Analysis, Including Estimating Shock Mitigation Effects of an Ideal Shock Isolator
- How SRS Analysis can be Distorted by Aliasing
- How SRS can be Over-Estimated by Insufficiently Damped Transient Simulations
- Making a Quick Estimate of Dominant Natural Frequencies in a Component
- Creating Shock Screening Protocols
- Highlights from *Exemplar Studies of Shock and PVSS*
- Additional Comments on Uses of SRS

Workshops

1. Transient Impact of a Container, Including SRS and PVSS Assessments
2. Using PVSS to Assess Which Shock is Worse for Which Payload

Appendix

1. Kornucopia® ML™ - DSP Features
2. Synthesizing a Realistic Transient Acceleration Shock Signal to Represent and Bound Diverse Oscillatory Shock Time-Histories
3. Kornucopia's Ability to Synthesize Realistic Oscillatory Transient Shock Signals, Including Statistical Considerations
4. Exemplar Studies of Shock and PVSS
   - Modified Scavuzzo Problem
   - Detailed Plate Shock Study

References

- Reference list and helpful web links

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