Enhancing Signal Accuracy and Bandwidth Extension in Transient Shock Measurements Using Transfer Function Compensation

Transfer Function Techniques for Improving Data Quality and Results Standardization

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Objectives

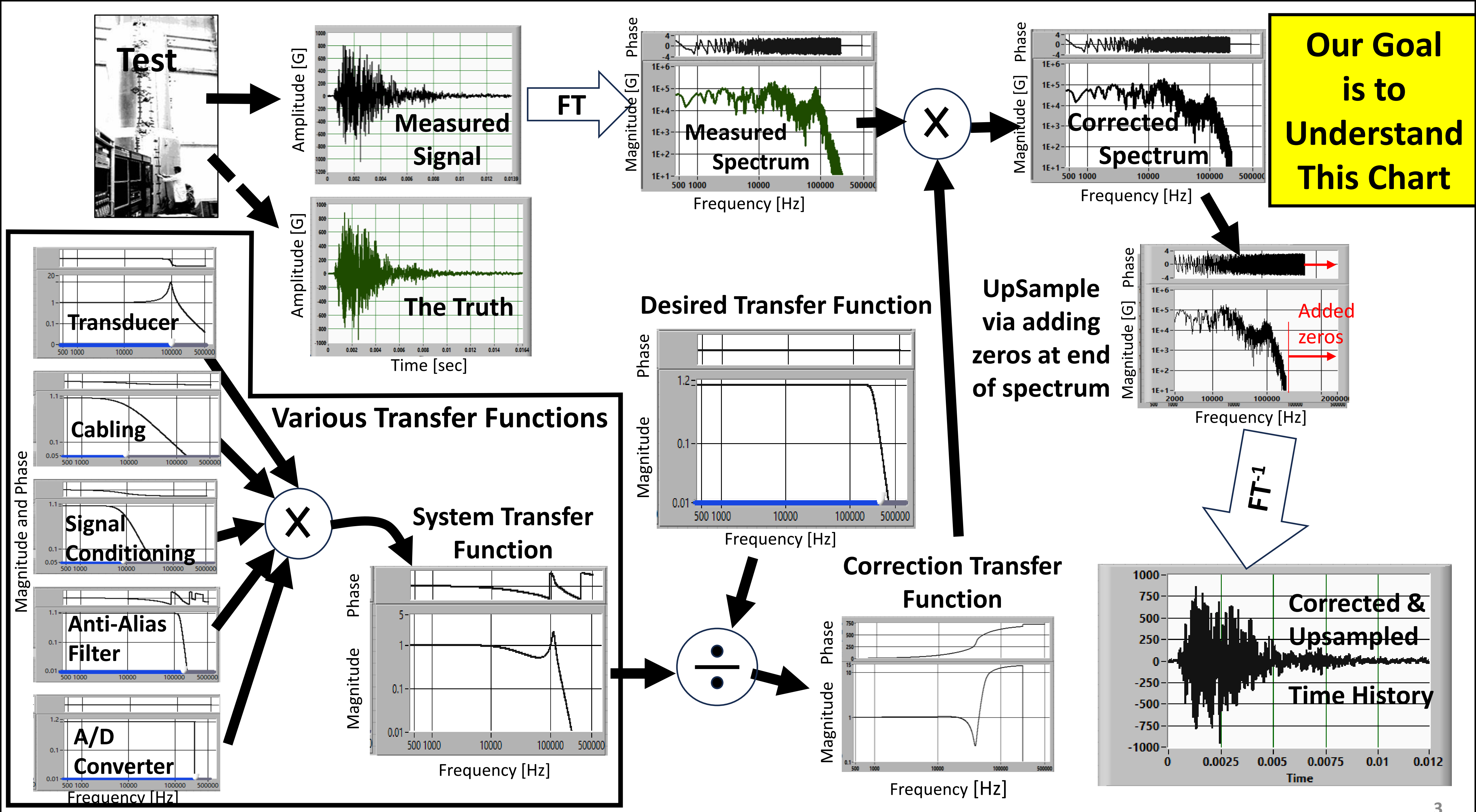
- Provide a Method for Removing the Distortions Introduced by the Non-Idealities of the Measurement Components.
- Increase the Frequency Range of the Measurement to Beyond Their Nominal Bandwidth.
- Demonstrate a “Normalization” Feature that Allows Results from Different Laboratories/Experiments to Be Directly Compared.

Background

Combines the Work of

- Walter
- PFI
- Smith

….Into One Integrated Digital Process.



Test
Amplitude [G]
Measured Signal
Amplitude [G]
The Truth
Time [sec]
FT
Phase
Magnitude [G]
Measured Spectrum
Frequency [Hz]
Phase
Magnitude [G]
Corrected Spectrum
Frequency [Hz]
Our Goal is to Understand This Chart
UpSample via adding zeros at end of spectrum
Phase
Magnitude [G]
Added zeros
Frequency [Hz]
FT-1
Corrected & Upsampled Time History
Time
Desired Transfer Function
Phase
Magnitude
Frequency [Hz]
Correction Transfer Function
Phase
Magnitude
Frequency [Hz]
Various Transfer Functions
Magnitude and Phase
Transducer
Cabling
Signal Conditioning
Anti-Alias Filter
A/D Converter
Frequency [Hz]
System Transfer Function
Phase
Magnitude
Frequency [Hz]

It Would Be Great If

All Test Labs Doing Similar Experiments

Used The Same

Transducers,

Cabling Systems,

Signal Conditioners,

Alias-Protection Devices and

Analog-to Digital Converters

and Data Acquisition Strategies

But They Don’t

So

Different Laboratories

Performing The

Same Experiment

Will Produce

Different Results

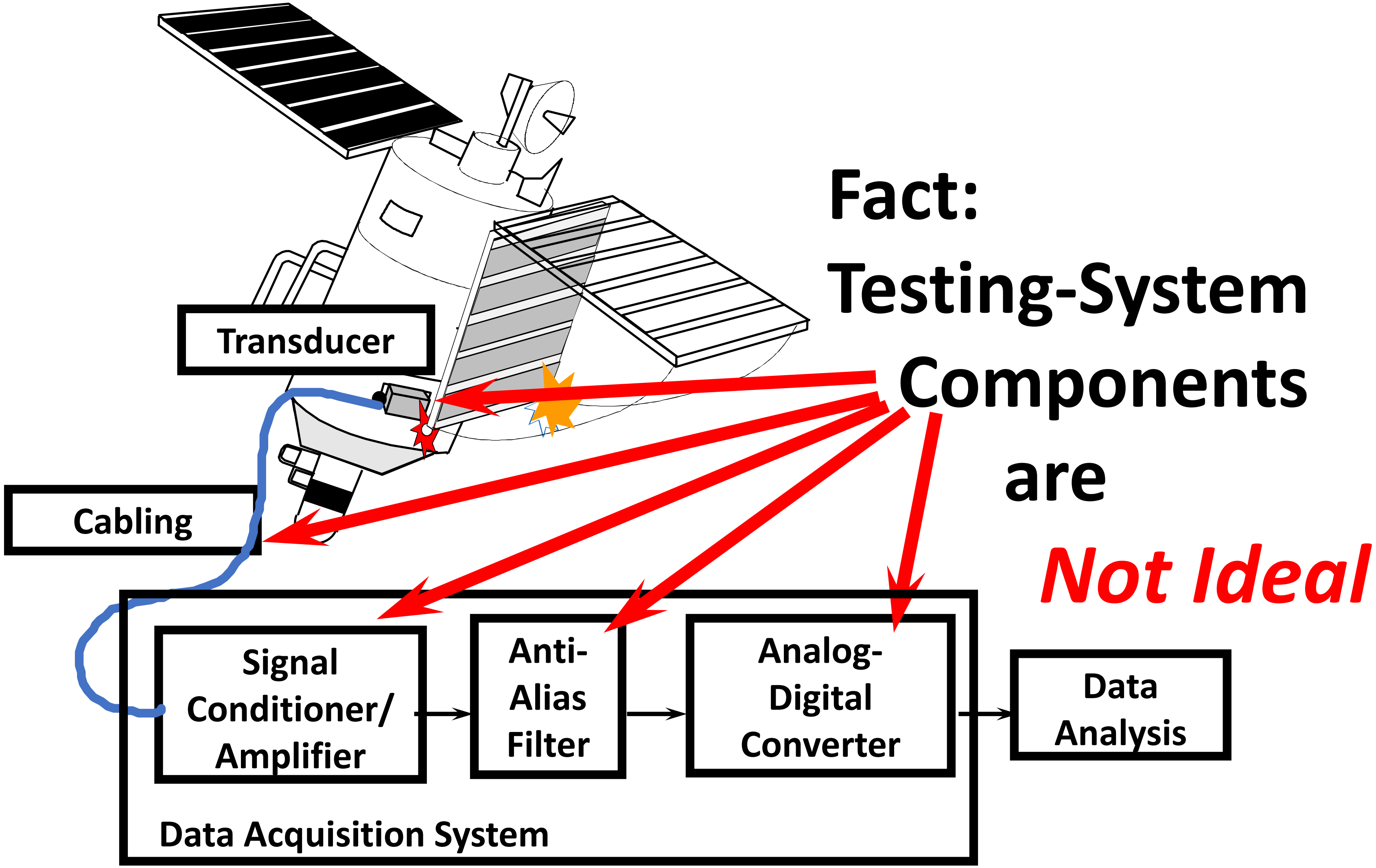
AND *(For Additional DSP-Related Reasons)*

Comparing Transient FEA To Physical Tests

Will Often Yield ***Different Results***

Why:

- Different Hardware Components
  In The Signal Chain
  Will Distort The Signal
  In Different Ways
- Different Measurement Bandwidth
  Will Produce Different Results


Fact:
Testing-System
Components
are
Not Ideal
Transducer
Cabling
Signal
Conditioner/
Amplifier
Anti-
Alias
Filter
Analog-
Digital
Converter
Data
Analysis
Data Acquisition System

Fact: A Real Measurement System
Will Always Produce
A Distorted
Version of the Truth

Our Objective:
Produce <u>Improved</u> Results With
Consistent Distortion (we cannot remove it all)
Between Tests/Laboratories (or Models and Tests)

Then, Results May Be Consistently Compared

Fact:
Transient FEA Models Also Produce A Distorted Version Of The Truth:
Physics + Solution noise + Modeling Approximations

History

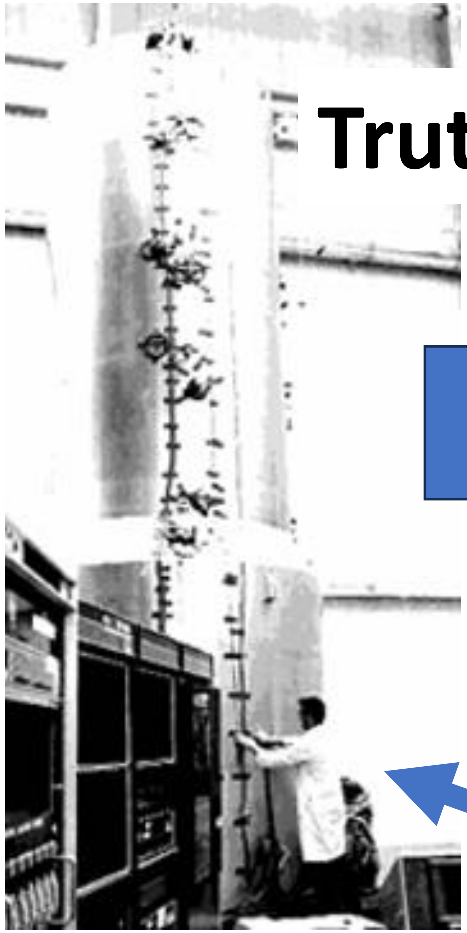
In 1981 Walter Proposed Using
Convolution Functions to Correct For The Distortion

In 1991 Smith & Hollowell Proposed Limiting
The Frequency Response of Shock Events
With a Standard Low-Pass Filter.

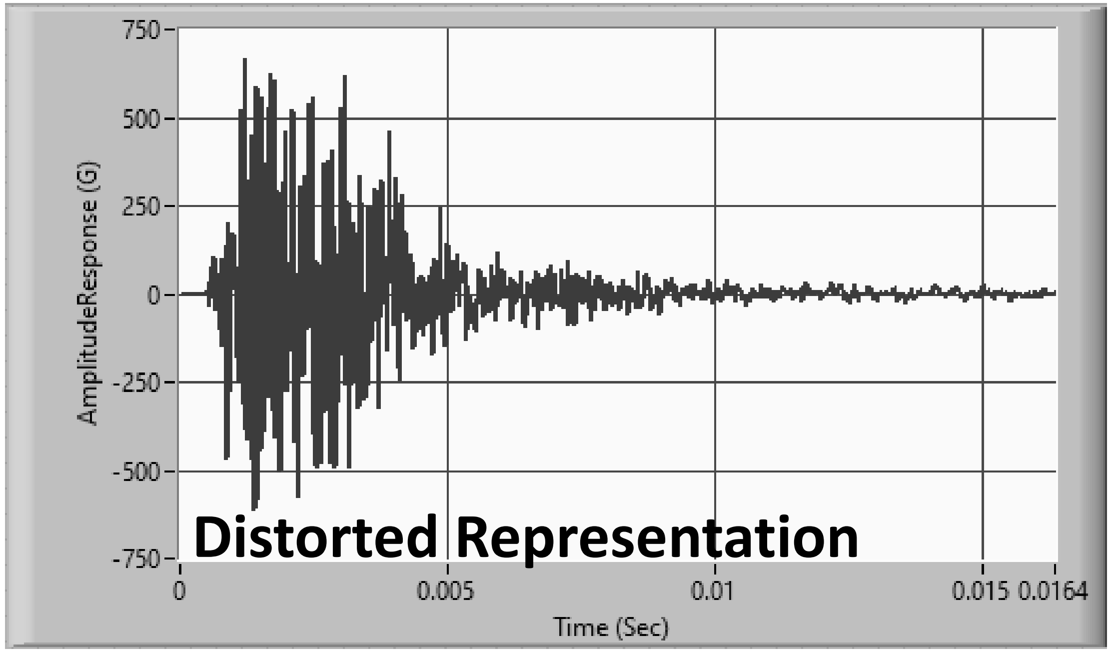
In 2022 Gerber, Firth, & Szary
Offered an Analog Approach For The Compensation Of
Transducer Resonance Effects

In This Presentation, the Concepts are
Combined In A Digital Process to Produce a
Consistently Distorted, Standardized, Result

Fact: Our Measurement Systems Do Not Tell Us The Truth



Measurement System

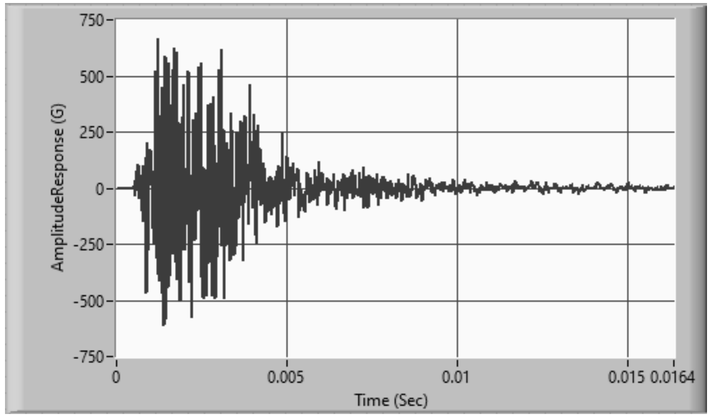


A Little History

Lockheed Sunnyvale's Hubble Space Telescope Demonstrator
(Lots of Firsts Accomplished With This Hardware and Test)

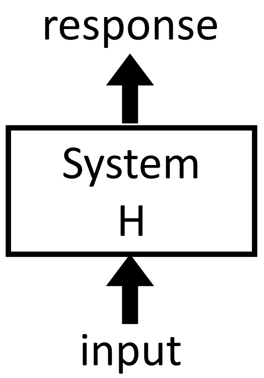
More Detail

Transducer → Cabling → Signal Conditioning → Anti-Alias Filter → A/D Converter



All Of The Components Contribute *Distortion*

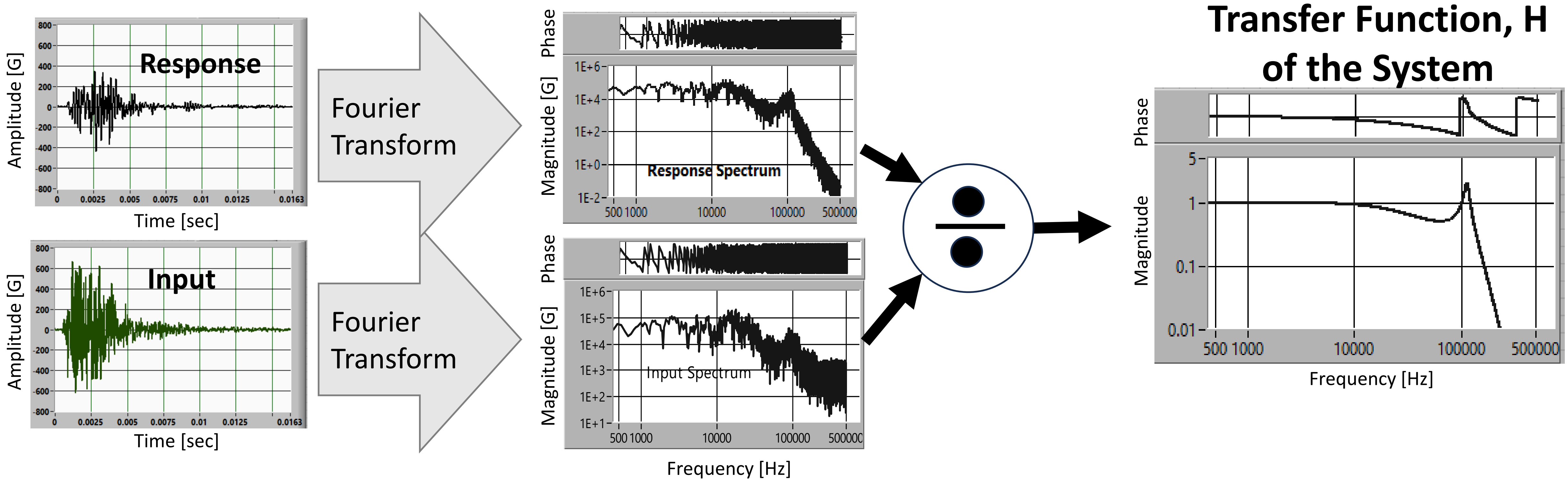
A Few Signal Analysis Basics



FT = Fourier Transform
FT^{-1} = Inverse Fourier Transform
H = TF = Transfer Function

For a Linear System, the Magnitude and Phase (delay) difference between Input and Response Signals can be Characterized With a Transfer Function.

The Basic Calculation Is:

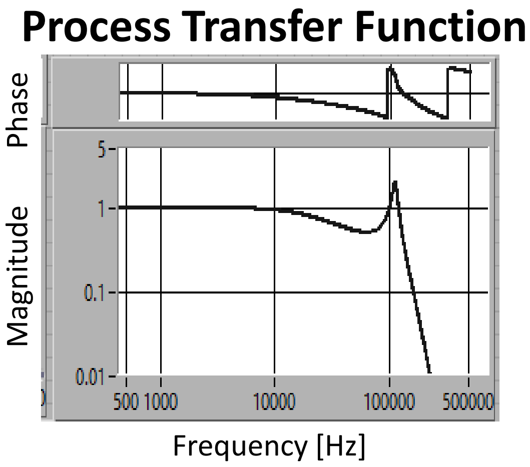


We Do Assume That The Elements Of Our Measurement System Are Linear!

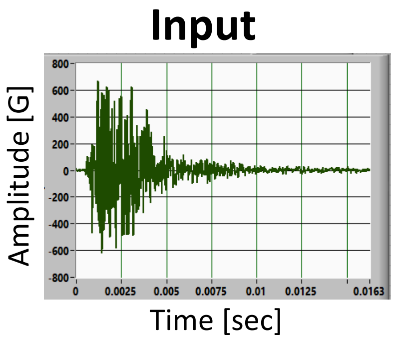
(and a few other assumptions to be noted shortly)

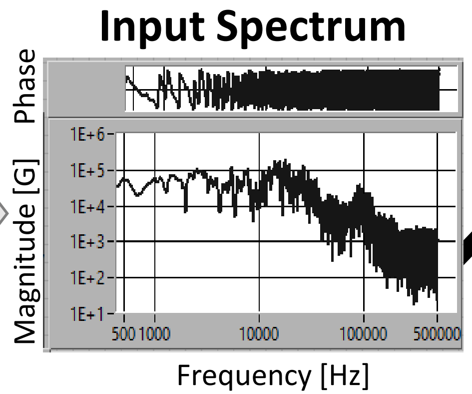
A Small Manipulation-Inverting The Process

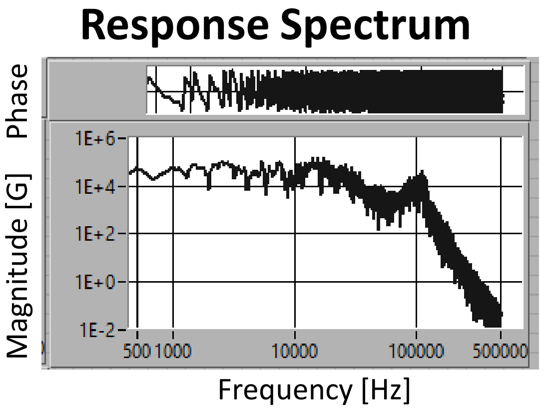
If We Know The Transfer Function Of A Process

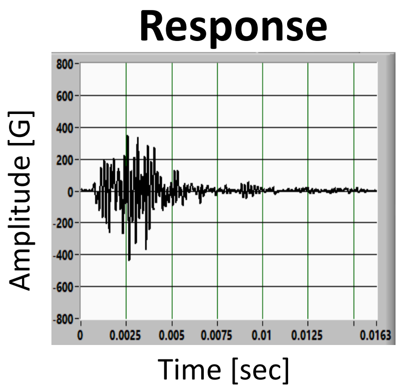


Then …









We Can Calculate The Response To Any Input**

** limited by the assumptions of the analysis (noted on next slide)

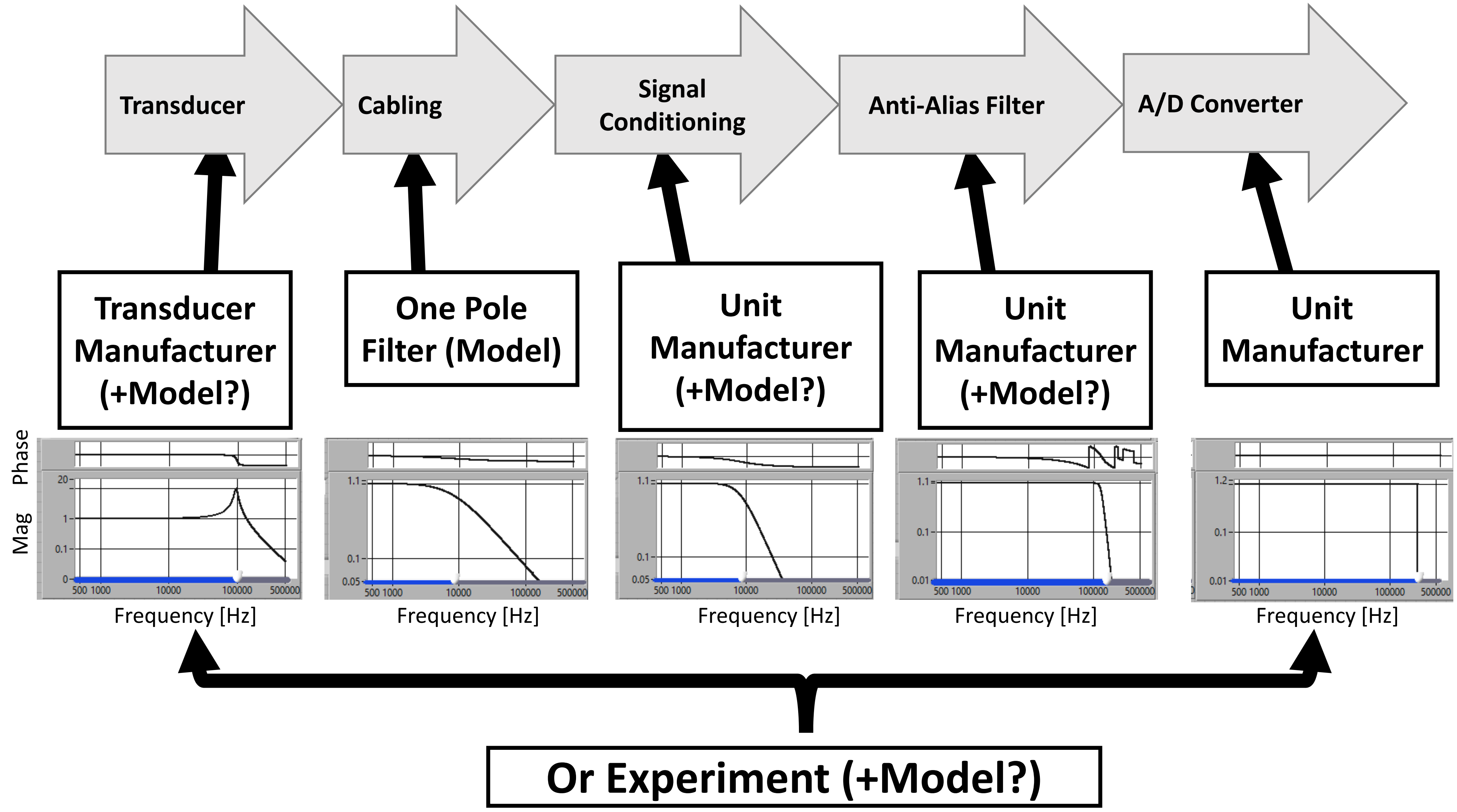
Additional Calculation Details

- All components within the system respond linearly.
- Fourier Analysis imposes assumption of *Periodic Signals* in calcs.
    - Many transient shocks approximate this assumption.
- Calculations presented use the unscaled Fourier kernel only.
    - Plots shown are magnitude and phase of 1st half of the Fourier kernel coefficients (2nd half are just mirror of complex conjugate).
    - For Fourier inverse calculations, most software will require you to provide the full spectrum, including the 2nd half of the coefficients.

$$\Psi_k = \sum_{n=0}^{N-1} y_n \, e^{\left(-jk\,2\pi\frac{n}{N}\right)}$$

- For typical shock/impact data, signals are short, so each spectrum is computed in 1 block (there is no multi-block averaging and no windowing like done in vibration analysis).
- Calculations such as multiplication or division of spectra are computed with the underlying spectrum’s complex coefficients (real + imaginary). The results of such math are complex coefficients too!

What Are The System Transfer Function Components?
……And, Where Do They Come From?



What Transfer Functions are Required.
…..And Where Do They Come From

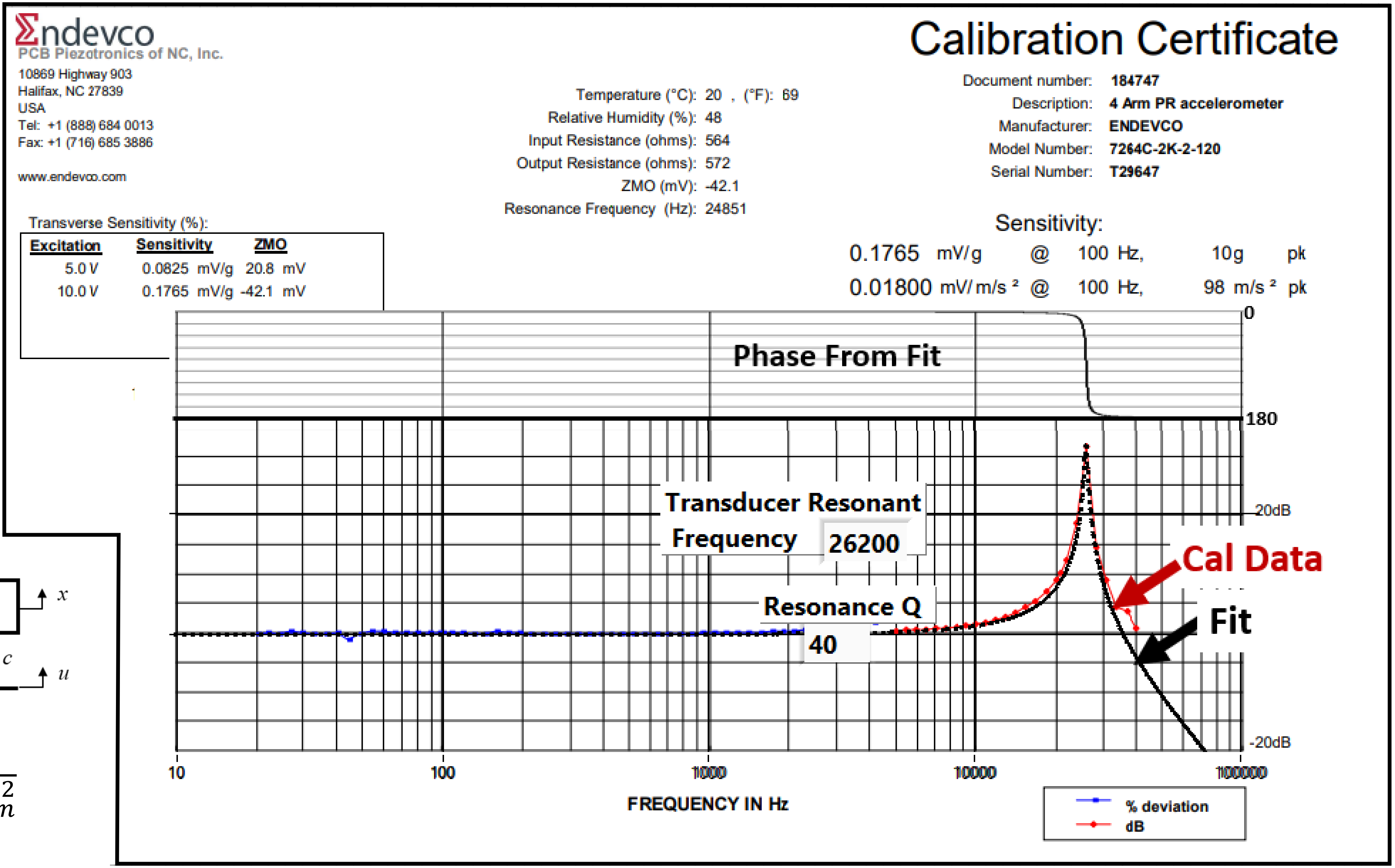
- Transducer Transfer Function
  - Normally Provided By The Transducer Vendor, BUT typically not up to the resonant frequency!
- Cabling
  - Normally a One Pole Filter With Cutoff Controlled By the Cable Capacitance, The Transducer Output Impedance, and Cable Resistance.
- Signal Conditioning
  - Normally a One or Two-Pole Filter Described by the Manufacturer.
- Anti-Alias Filter
  - Characteristics From The Hardware Vendor
- A/D Converter
  - Characteristics From The Hardware Vendor

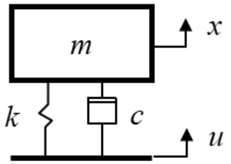
OR
May Be These Should Be Characterized Experimentally ???

Special Calibration Provided By Transducer Vendor At Our Request

Red Curve Is Measurement By Vendor.

Black Curves (including Phase) Are Fit by Us Using SDOF Model





$$H(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

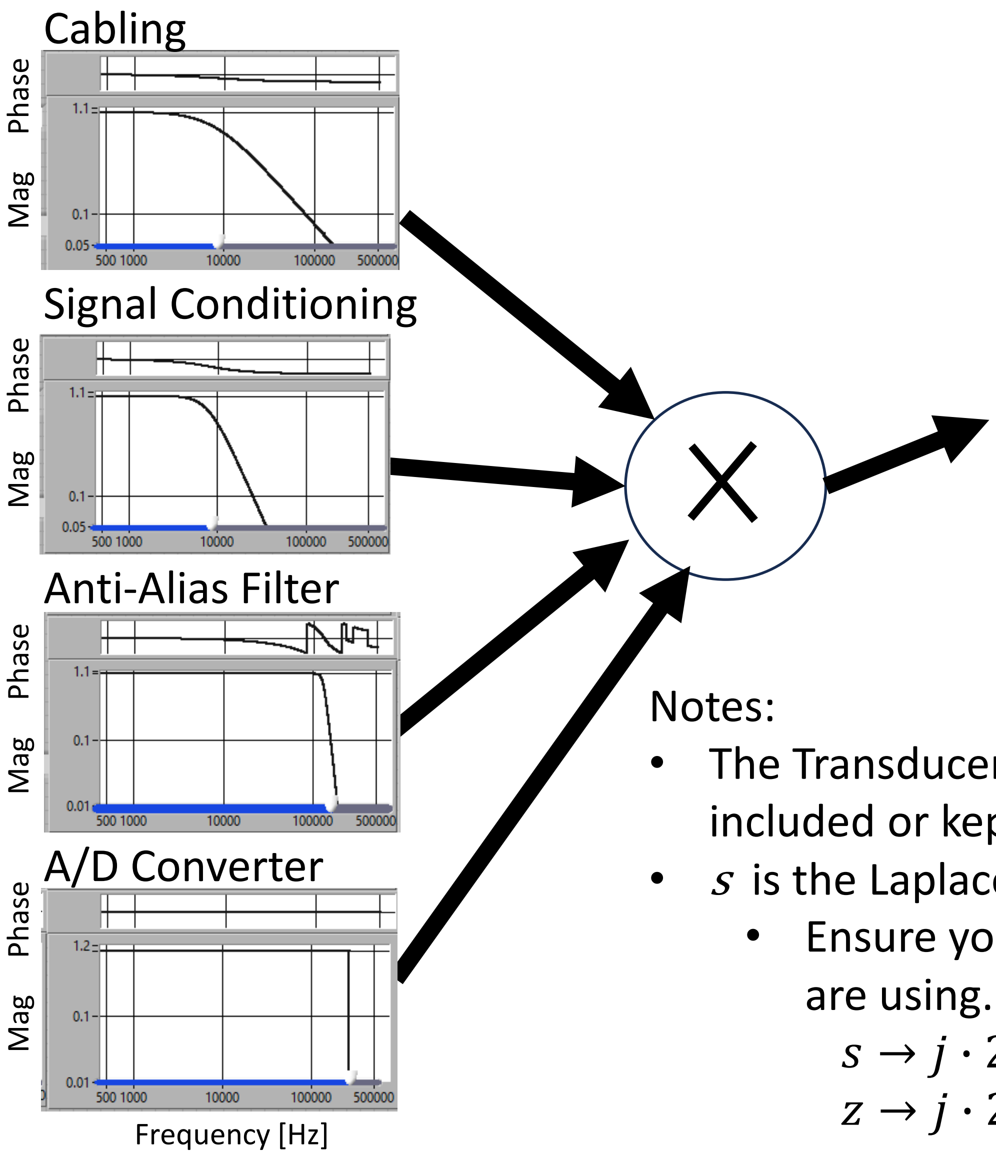
Analytically Modeling The Data System Transfer Function (Excluding the Transducer)

$$H(s) = \frac{1}{1 + s \cdot R \cdot C}$$

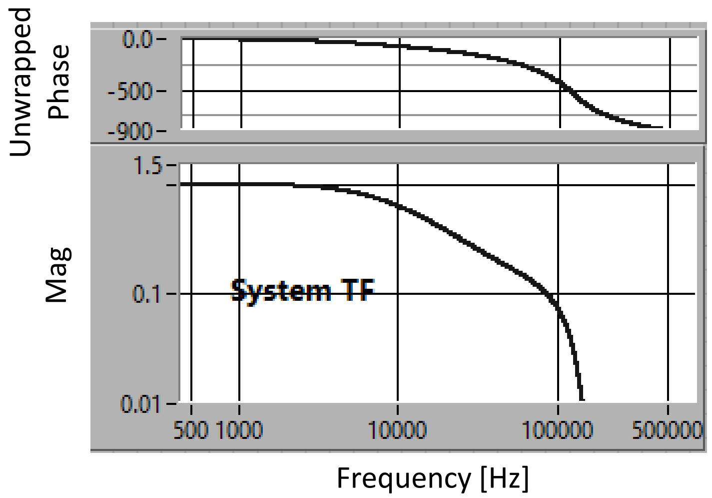
R, C are cable DC resistance and Cond-to-Cond capacitance

$$H(z) = \frac{B_0 + B_1 z^{-1} + \cdots + B_L z^{-L}}{1 + A_1 z^{-1} + \cdots + A_L z^{-L}}$$

A & B values are filter coefficients



System Transfer Function (Both Phase & Magnitude)



Notes:

- The Transducer is NOT represented here but it could be included or kept separate.
- s is the Laplace domain, z is the digital domain
  - Ensure you understand which type of TF form you are using. You evaluate them differently.

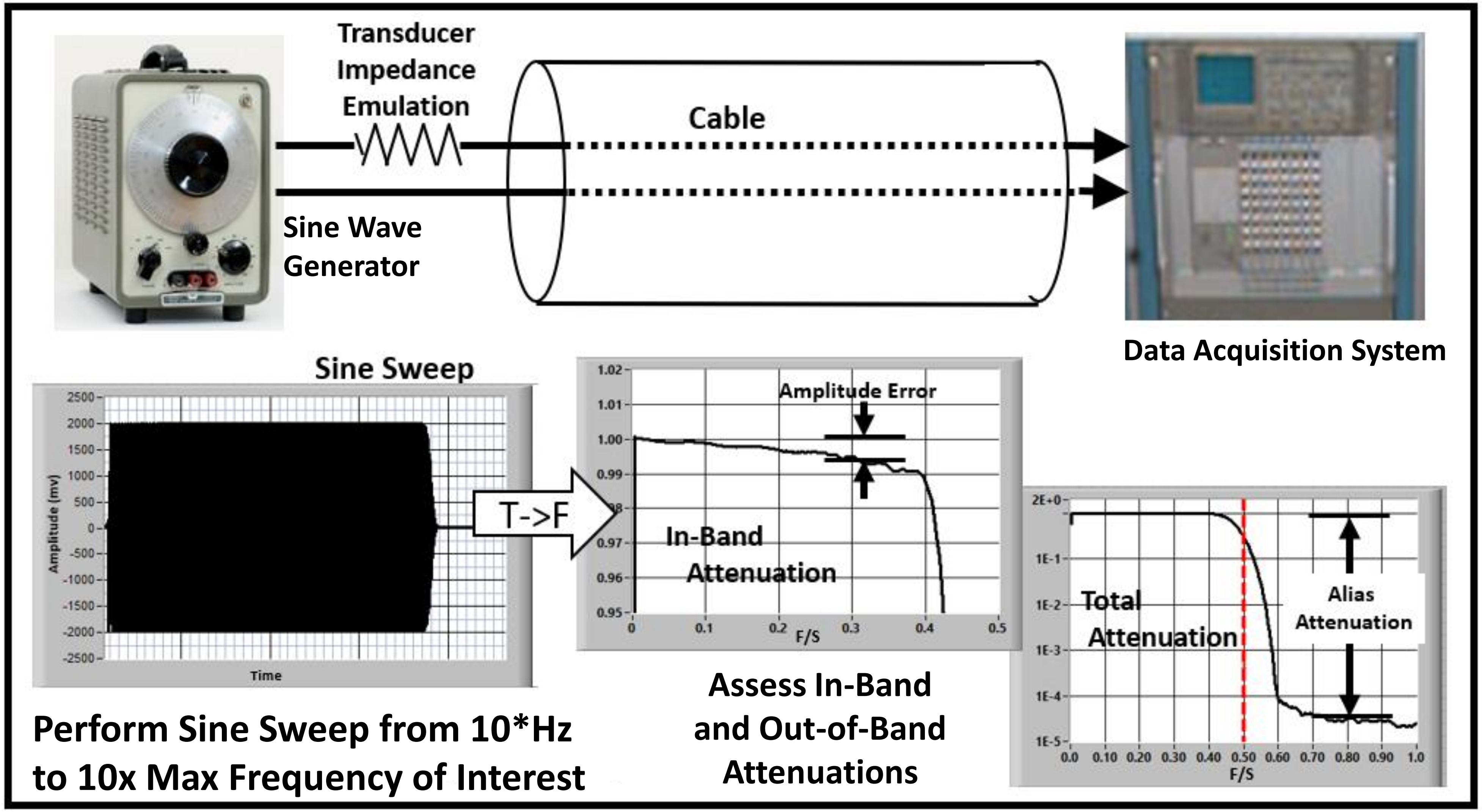
$$s \rightarrow j \cdot 2 \cdot \pi \cdot f$$
$$z \rightarrow j \cdot 2 \cdot \pi \cdot f / f_s$$

Experimental Characterization Of The System Transfer Function (Excluding Transducer**)

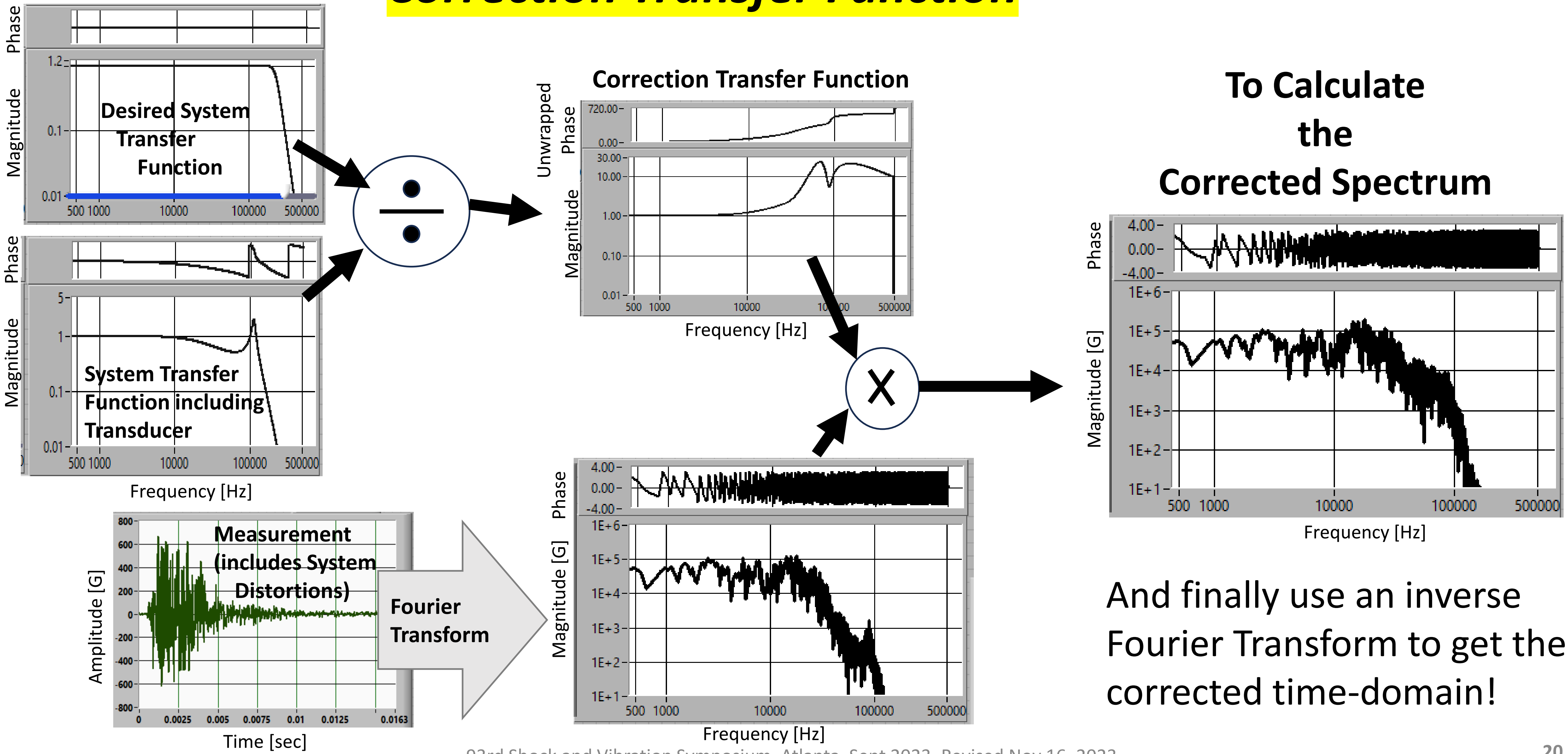
** The electrical impedance of the transducer is included but not its full transfer function.

For a nice reference discussing this approach, see:

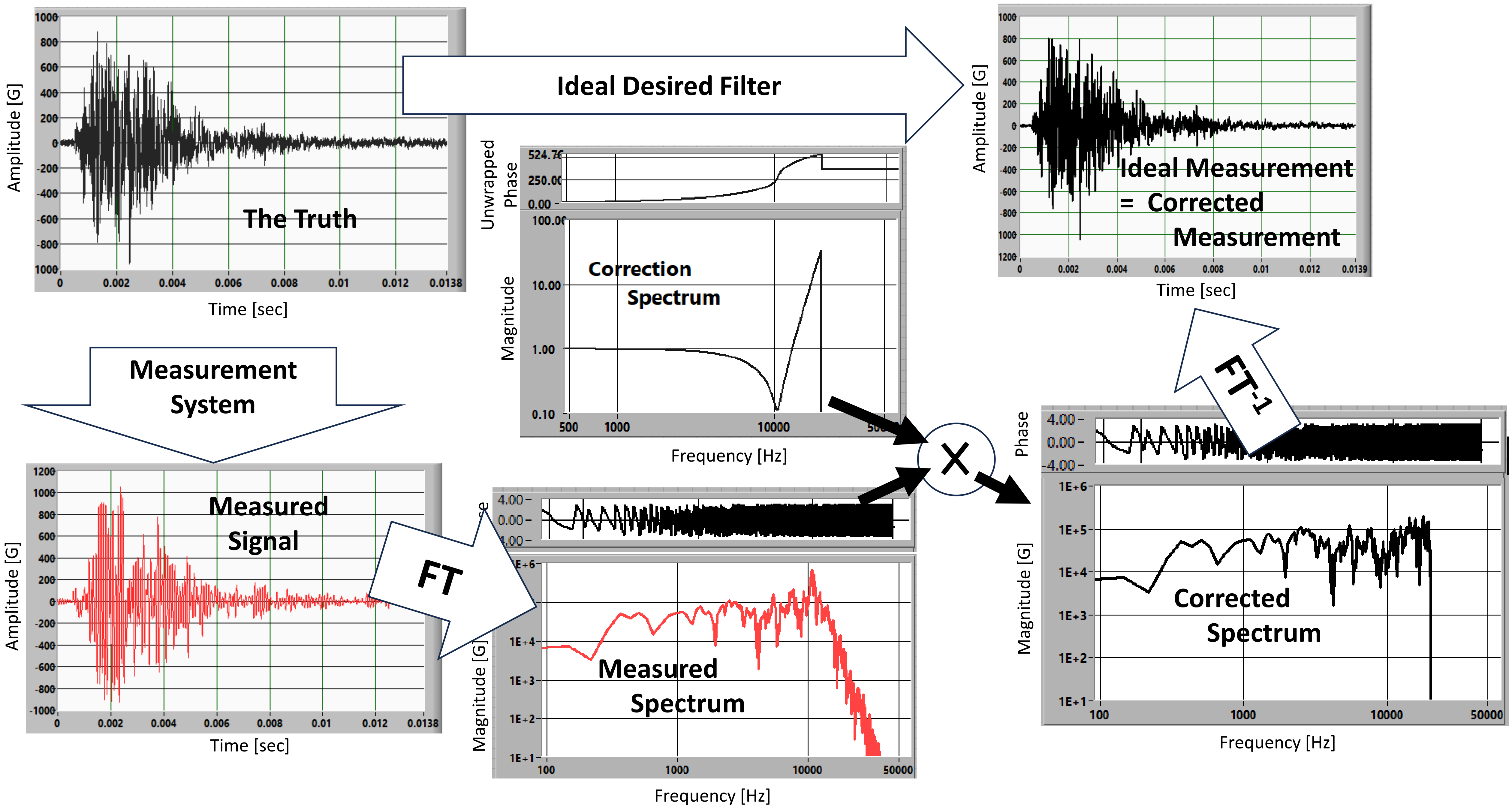
MEMS Shock Accelerometer Signal Modification Attributable To The Electrical Impedance Of Their Cables
Patrick Walter, Alan Szary, James Woernley, 2021,
Find at www.pcb.com



Then We Apply a Desired System Characteristic to Get a *Correction Transfer Function*



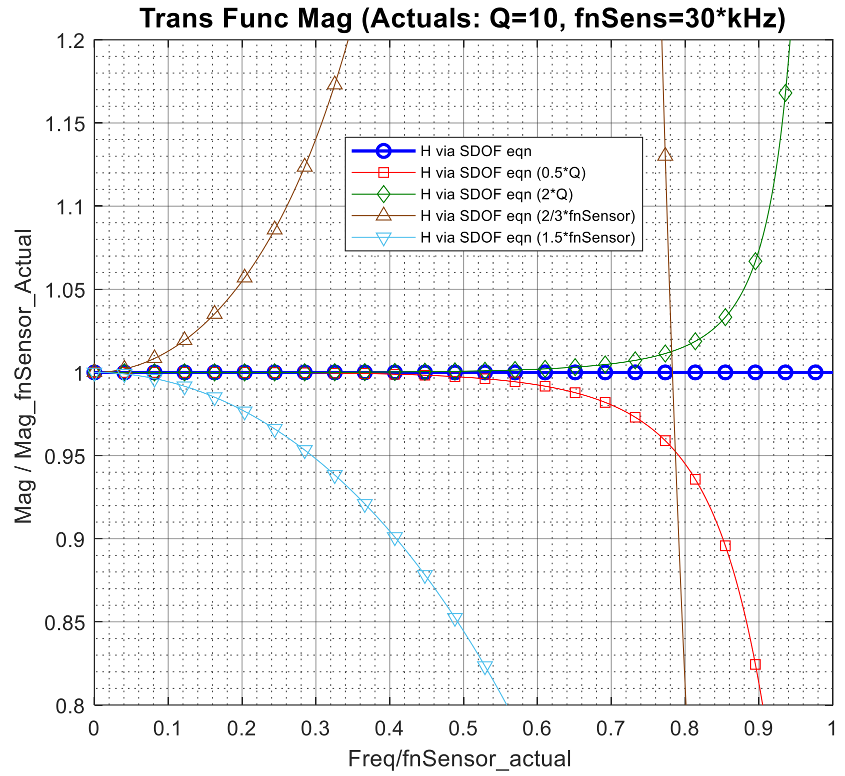
And finally use an inverse Fourier Transform to get the corrected time-domain!


The Truth
Amplitude [G]
Time [sec]
Ideal Desired Filter
Unwrapped Phase
Magnitude
Correction Spectrum
Frequency [Hz]
Ideal Measurement = Corrected Measurement
Measurement System
Measured Signal
FT
Measured Spectrum
Magnitude [G]
Phase
Corrected Spectrum
FT-1

Limitations on Transfer Function Corrections

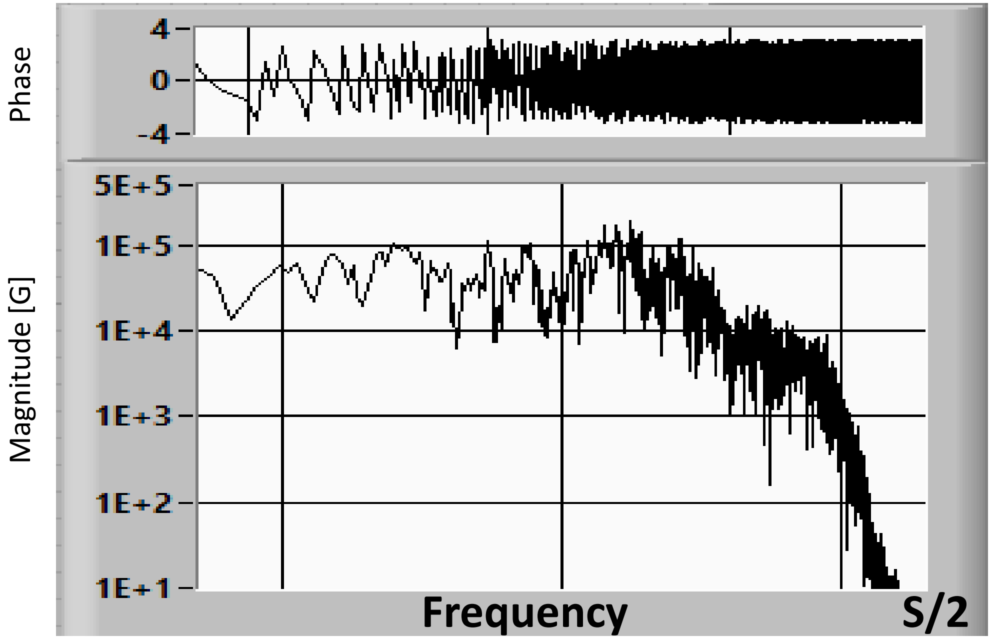
The following realities will limit how much of a correction you can make.

- Discrepancies between TF from analytical models compared to actual hardware. Additionally, uncertainty in parameters such as sensor resonance and sensor damping, or cabling capacitance and impedance.
- Numerical issues trying to revive frequency content squelched by low pass filtering in components of measurement chain.
  - Tip: Keeping flat pre-trigger data section in analysis helps judge when you are pushing correction to too high a frequency (you will see noise rise-up in the pre-trigger section).



Above plot shows accuracy of TF correction for ideal accelerometer when fn and Q of sensor have plausible uncertainties. TF most sensitive to fn sensor, less sensitive Q of sensor.

We Can Upsample Via Spectrum Padding With Zeroes



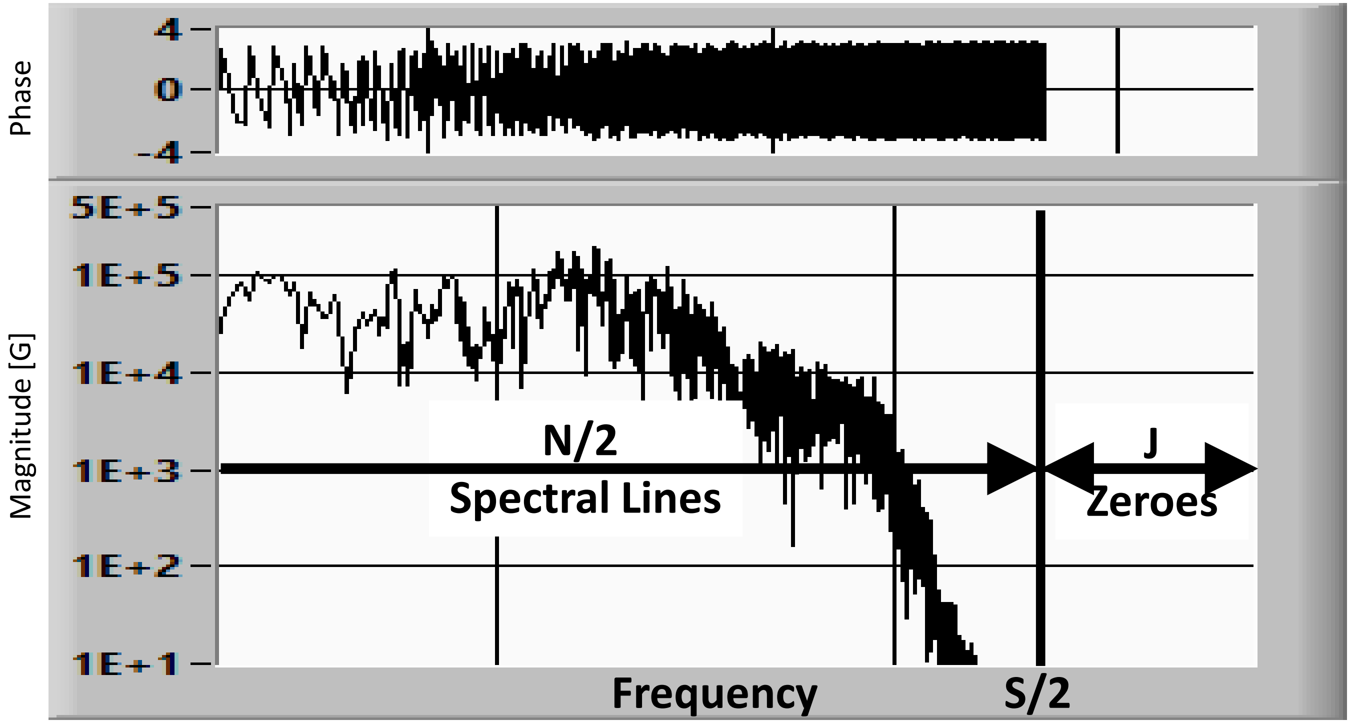
Increasing the number of Spectral Lines by Padding Produces More Time History Points

Upsample Ratio=(N/2 + J)/(N/2)

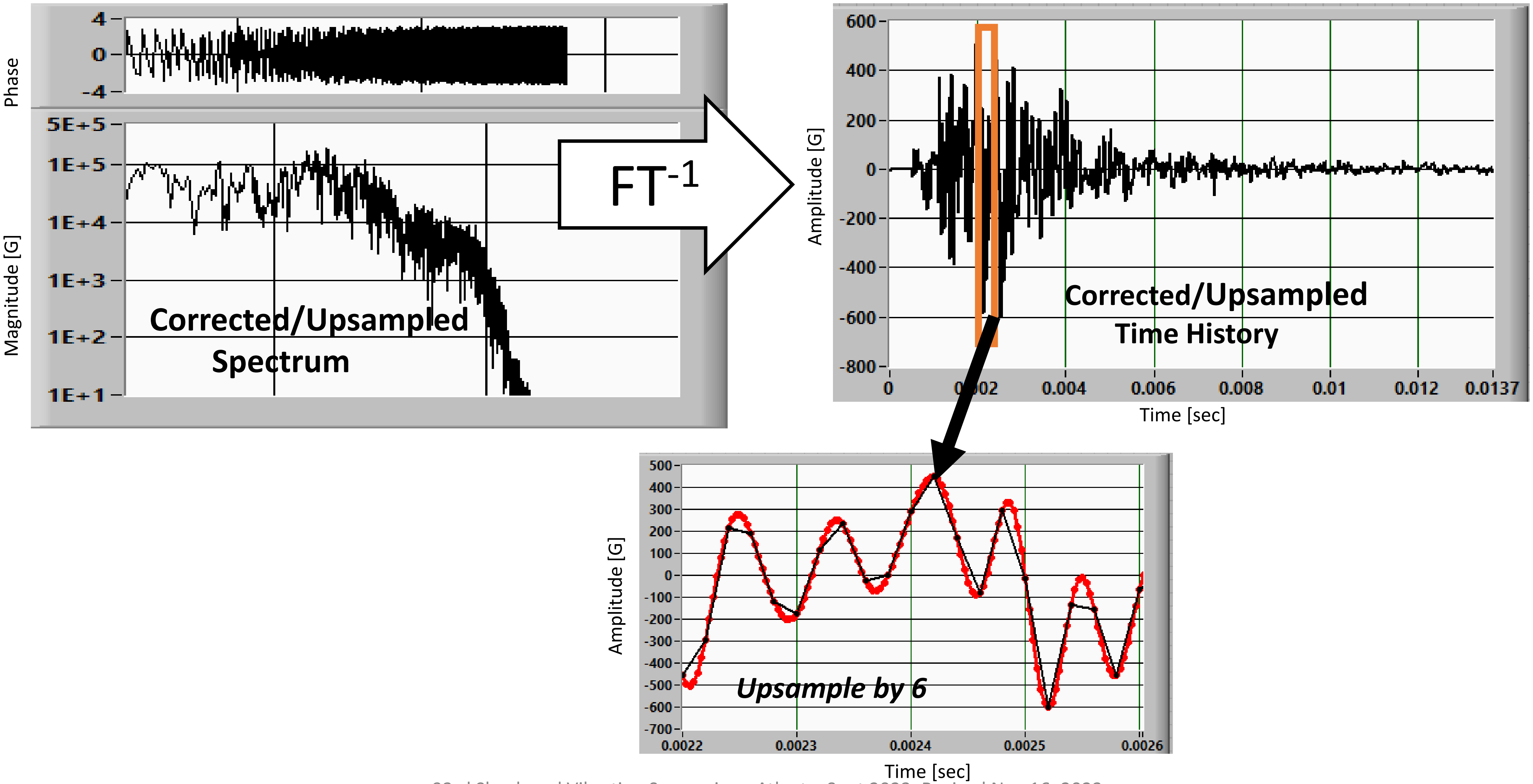
S=Sample Rate

N=Number of points in the Original Time History

J=Number of "Padding" Zeroes



Finally, We Convert back to the Time Domain



Conclusions

- All Measurement Systems Record A Distorted Version Of The Truth.
  - Various Components In The System Put In Different Types Of Distortions.
- Using Fourier Analysis, We Can Create Transfer Functions To Represent Various Components Of The System As Well As The Entire System.
  - Do This With Theoretical Models, Measurements, Or Some Combination.
- We Can Therefore Create A Correction Transform That Attempts To Nullify The Measurement's Distortions.
  - There Are Limitations To The Amount Of Correction That Can Be Made.
  - You Should Do Diligent Sanity Checking Of Any Such Corrections That Are Made.
- These Techniques Allows For Improved And More Consistent Results Comparisons From Different Laboratories/Experiments (Or Between Transient FEA Simulations And Physical Tests).

References

1. ***Deconvolution as a Technique to Improve Measurement-System Data Integrity***
   Patrick Walter, Experimental Mechanics August 1981

2. ***Techniques For The Normalization OF Shock Data***
   Strether Smith and William Hollowell,
   62nd Shock and Vibration Symposium (October 1991)

3. ***Frequency Response Compensation for Resonant Sensors***
   Thomas P. Gerber, Douglas R. Firth, & Alan R. Szary
   Precision Filters 2022

4. ***MEMS Shock Accelerometer Signal Modification Attributable To The Electrical Impedance Of Their Cables***
   Patrick Walter, Alan Szary, James Woernley 2021
   Find at www.pcb.com